Tax-Induced Barriers to Female Workers in Japan: Insights from Dynamic Intra-Household Bargaining*

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Preliminary Version

The latest version is available [HERE].

Abstract

Many married women in Japan face a distorting tax system called "Income barriers," which discourages them from working by penalizing earnings above certain thresholds. While the negative impacts on labor supply have been empirically studied, the dynamic and welfare consequences remain underexplored. This paper quantifies the impacts using a heterogeneous-agent life-cycle model. The model explicitly incorporates intra-household bargaining between spouses — a feature often ignored in macroeconomic literature. Estimation with the Japanese Panel Survey of Consumers (JPSC) reveals that Income barriers reduce married women's labor supply by 20.4%, and this figure would be underestimated if intra-household bargaining is ignored. Also, a revenue-neutral elimination of Income barriers using Earned Income Tax Credit (EITC) increases welfare by 2.1%. These findings underscore both the sizable welfare losses caused by Income barriers and the importance of modeling intra-household bargaining in policy evaluations. The framework of intra-household bargaining developed here is applicable to other policy contexts.

Keywords: Labor supply, intra-household bargaining, tax and social security, heterogeneous agents, life-cycle, Japan

JEL Codes: C78, D15, H2, H31, J22

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1 Introduction

If you earn more, you gain more — self-evident, and no one would deny it. However, this is not always the case with couples in Japan. They face "Income barriers"; if their earnings exceed certain thresholds, they are penalized by additional tax burdens. As a result, their after-tax income decreases, although the before-tax income increases.

Japan has several Income barriers, and the most notorious one is the "1.3 million yen barrier." If a wife[1] earns more than JPY 1.3 million annually, her after-tax income decreases by JPY 200,000-300,000. How distorting are such barriers? According to the General Survey on Part-time Workers by the Ministry of Health, Labour and Welfare (MHLW) in 2021, 21.8% of married female part-time workers suppress their working hours to avoid the barriers.

This paper investigates the impacts of Income barriers focusing on welfare. Although a wide range of existing studies agree on their significant negative impacts on married female labor supply, the suppressed labor supply does not necessarily mean a welfare loss because working involves disutility. To determine whether and how Income barriers cause welfare losses, a dynamic structural model is constructed. The model is populated by agents with life-cycles and heterogeneous in sex, education, marital and parenting status, and productivity. These features capture the various aspects of Income barriers, such as the effects of suppressed working hours on skill formation. Also, the model contains the labor market and can analyze the impacts through the labor demand. The model is estimated using the Japanese Panel Survey of Consumers (JPSC), and counterfactual experiments are conducted to quantify the impacts of Income barriers.

The model's most unique and essential feature is its explicit modeling of intra-household bargaining, which macroeconomic studies often ignore. A household is no longer a single decision-maker; a married couple allocates consumption, leisure, and housework by bargaining. The contribution of intra-household bargaining is summarized by a mechanism called *bargaining effect*. Bargaining effect is an effect of intra-household bargaining on allocation within a household. If a spouse would be better off after divorce than the other spouse, s/he less fears the divorce; this enables her/him to make stronger claims and obtain a larger share of the household's consumption. While this intra-household bargaining is ignored in most macroeconomic studies, it can cause a biased result if ignored. For example, an increase in wife's wage would increase her labor supply, but at the same time, it also increases her bargaining power. As a result, she obtains more consumption and leisure, which negatively affects her labor supply (Figure 1). By incorporating the intra-household bargaining, this paper tackles the bias and suggests the importance of intra-household bargaining in policy evaluations.

[1] Precisely, a secondary earner of a couple whose spouse is employed and insured by the Employees' Pension Plan. Since around 90% of the second earners of married couples in Japan are wives (Labour Force Survey), I call the secondary earner "wife" for clarity without any gender discriminative intentions.

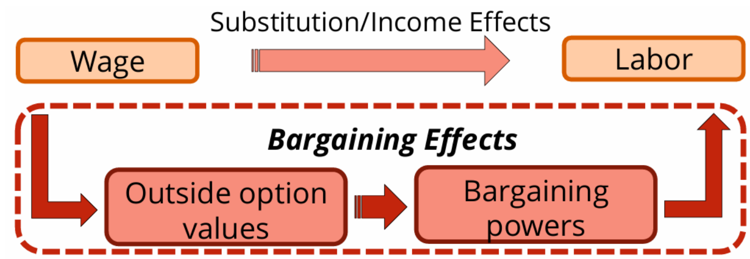


Figure 1: Bargaining Effect

The main results of the paper are summarized as follows. The removal of the 1.3 million yen and other barriers raises married women's labor supply by 20.4%. However, since the female wage declines due to the increase in the female labor supply, the effect on extensive margin is opposite to intensive margin; the labor force participation declines by 1.4% for married women. Also, if the intra-household bargaining is ignored, the increase in the labor supply would be underestimated. Regarding welfare, a simple removal of barriers causes welfare loss because it increases the tax burden. Besides, the welfare loss is biased towards wives rather than husbands. However, if a revenue-neutral Earned Income Tax Credit (EITC) is properly introduced instead of removing all barriers, the welfare increases by 2.1%.

The contributions of this paper are twofold. The first is to the literature on Income barriers. There is already a large body of literature on Income barriers, but most are event studies using policy changes as natural experiments. Besides, they focus only on the labor supply and static effects and ignores the effects on labor market. This paper constructs a structural dynamic model with labor market, which provides richer counterfactual experiments and analyses of the persistent effects on welfare. The second is to family-macroeconomic literature. Using simple models, this paper formulates the bargaining effect and shows the importance of intra-household bargaining in evaluating general policies. It also formulates a general framework of bargaining structure called "dynamic Nash bargaining without commitment," which can be applied to many other policy evaluations.

This paper is organized as follows. The remainder of the section 1 reviews the related literature. Section 2 explains the institutional backgrounds of Income barriers. Section 3 sets up the theoretical frameworks of intra-household bargaining, while section 4 constructs a full model for quantitative analysis. The calibration and quantitative results are shown in section 5 and section 6, respectively. Section 7 concludes the paper.

Related Literature This paper stands on three strands of literature. The first is the literature on Income barriers. There exist a number of empirical studies, and most agree on the significant negative impacts of Income barriers on married female labor supply. Sakata and McKenzie (2005), Yokoyama (2018), and Kondo and Fukai (2023) estimate the effect by with statistical approaches.

They use policy reforms, in which the thresholds of the barriers are changed, as instruments and find the decrease in the labor supply of those confronted with the barriers. The representative studies with structural models are Akabayashi (2006) and Bessho and Hayashi (2014). They estimate microeconomic models of a married couple and conduct counterfactual analyses. Both find the negative effect on wives' labor supply, and Akabayashi (2006) further points out the inefficiency of intra-household resource allocation caused by Income barriers. Studies that analyze dynamic or welfare effects are limited. Yamada (2011) constructs an overlapping generations model and estimates a model-based regression with a policy reform as an instrument. Kitao and Mikoshiba (2022), which is most closely related to this paper, construct a life-cycle model with heterogeneous agents. They conduct counterfactual analysis and find the negative impacts both on labor supply and welfare.

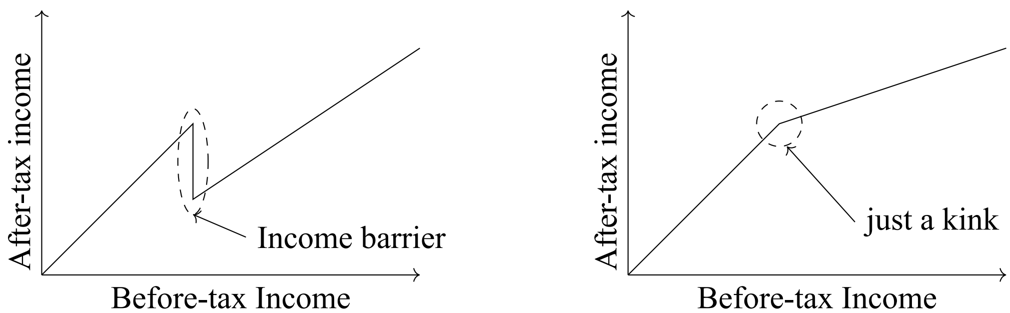
The second is the literature on intra-household bargaining. Chiappori (1988) and Chiappori (1992) build a theoretical framework for intra-household bargaining and formulate the importance of bargaining power over resource allocation. Lundberg et al. (1997), Ward-Batts (2008), and Attanasio and Lechene (2002) empirically conclude that the bargaining power of each spouse is not constant but varies with the economic environment, such as the relative wage. While studies above focus only on static effects, Voena (2015), Chiappori et al. (2017), Guo and Xie (2024), and Lise and Yamada (2019) are based on dynamic settings. Voena (2015) finds that the divorce law reform, which affects the outside option values, significantly changes women's working and saving behavior. Lise and Yamada (2019) construct a life-cycle model and estimate it with JPSC data. They find that each spouse's bargaining power is not constant but varies mainly depending on their relative wage. They also conclude that the commitment between spouses is not perfect. Guo and Xie (2024) also use JPSC data and find that childbirths significantly reduce wives' bargaining power. This paper extends the empirical findings of these studies (varying bargaining power and limited commitment between spouses) to the analysis of Income barriers.

The third is the literature on family-macroeconomics which analyzes female labor supply. A wide range of studies exists that evaluate the impacts of policies on female labor supply from macroeconomic perspectives. Borella et al. (2023), Kitao and Mikoshiba (2022), and Guner et al. (2023) belong to this strand and closely related to this paper. All of them construct a life-cycle model with heterogeneous agents, and while Kitao and Mikoshiba (2022) use Japanese data, Borella et al. (2023) and Guner et al. (2023) use the US data. Borella et al. (2023) analyze the impacts of marital-status-dependent tax systems on female labor supply and welfare. Guner et al. (2023) construct a model with general equilibrium and estimate the impacts of introducing a flat tax system. They model the income process of married couples and the childcare costs in detail and find that the flat tax system increases the married female labor supply. However, there is no family-economic study with explicit intra-household bargaining.

2 Institutional Background of Income Barriers

2.1 Descriptions

While there is no widely accepted definition of Income barriers, this paper defines them as “the income thresholds at which a household’s marginal after-tax income becomes negative.” There are two important points. First, at Income barriers, the marginal after-tax income of a household is negative (the left panel of Figure 2) due to sudden increases in tax burdens or social security contributions. With a progressive tax, it is natural that some kinks exist in after-tax income like the right panel. However, such declines in after-tax income are not common in other countries.



(a) Income barrier　　**(b)** *NOT* Income barrier

Figure 2: Definition of Income Barriers

Second, this paper focuses on a *household* income, not an *individual* income, especially the joint income of a married couple. Basically, Japan’s tax unit is an individual, not a household. However, some taxes and social security contributions depend on the family structure. To capture the distortion coming from these family-dependent tax systems, this paper focuses on household income.

2.2 Sources and Examples

Table 1 exemplifies Income barriers. A detailed explanation is provided in the appendix D; thus, only a brief description is presented here. There are two main sources: income taxes and social security contributions. The representative example is spousal deductions in income tax. If the secondary earner within a couple (usually a wife) earns less than JPY 1.5 million annually, s/he is *dependent* on her/his spouse. Then, the primary earner (usually a husband) can call for an income deduction of JPY 380,000, which reduces income tax by the tax rate $\times$ JPY 380,000. The deduction amount gradually decreases as the secondary earner’s income increases. As a result, if the primary earner’s tax burden increases and the after-tax income decreases.

Regarding social security contributions, there are some thresholds below which the contributions are exempted. For example, if a primary earner is employed and the secondary earner's annual income is less than JPY 1.3 million, the secondary earner is called *Category III insured*: s/he is exempted from paying the contribution of the National Pension[2] while s/he can still receive the pension benefits in the future. In addition, s/he is also exempted from paying the Health Insurance if the annual income is below JPY 1.3 million. These exemptions can also create Income barriers called the 1.3 million yen barrier, which causes about JPY 200,000 - 300,000 of after-tax income loss for the couple.

Table 1: Examples of Income Barriers[3]

Name	Threshold (JPY)	Magnitude (JPY)
Income tax		
Local income tax	about 1 mil.	about 5,000
Spousal deduction	1.5 mil.	prim. earner's tax rate$\times$20,000
Social securities		
Employees' Pension[4]	1.06 mil.[5]	100,000 - 200,000
Category III of National Pension[6]	1.3 mil.	about 200,000
Health insurance[6]	1.3 mil.	about 100,000

Note: In 2024. The barriers shown are not exhaustive and change over time. Appendix D gives more comprehensive description.

3 Theoretical Frameworks of Intra-Household Bargaining

This section presents the intuitions and the importance of intra-household bargaining in policy evaluations. First, I compare two theoretical frameworks of intra-household bargaining: a unitary model and a collective model, referring to Chiappori (1992). Using a static model, I show that the existence of the *bargaining effect* can substantially affect the intra-household allocation of time and consumption. Next, I extend the model further to a general dynamic setting and present a rationale and a motivation for the full model in the next section.

[2] Japan has two main public pension plans: the National Pension Plan and the Employees' Pension Plan.
[3] I partially refer to Table 1 in Kondo and Fukai (2023).
[4] Precisely, this is not necessarily a barrier in lifetime (see appendix D).
[5] The eligibility depends on the firm size and weekly hours worked.
[6] Only for spouses of Category II insurees.

3.1 Static Model

Preferences and Constraints A household consists of a husband H and a wife W. They have utility from consumption and leisure: $U^i = U^i(c_i, l_i)$, where c_i is consumption and l_i is leisure for $i \in \{H, W\}$. Suppose U^i has the standard properties: U^i is twice continuously differentiable, strictly increasing, and strictly concave in each argument. A husband and a wife are endowed with a unit of time, respectively, and allocate it into working and leisure: $1 \geq l_i + h_i$, where h_i is working hours. While a couple are subject to time constraints separately, they share the budget and the same constraint: $c_H + c_W = w_H h_H + w_W h_W$, where w_i is the wage rate of $i \in \{H, W\}$.

Unitary Model vs. Collective Model One of the simplest ways to formulate a decision-making of a couple is as follows:

Definition 1 (Almås et al. (2023))**.** The unitary model is a model under which a couple maximizes the sum of their private utilities with a constant weight μ; for example,

$$\max_{c_H, c_W, h_H, h_W, l_H, l_W} \mu U^H(c_H, l_H) + (1-\mu) U^W(c_W, l_W), \tag{1}$$
$$\text{s.t. } c_H + c_W \leq w_H h_H + w_W h_W, \quad 1 \geq l_H + h_H, \quad 1 \geq l_W + h_W.$$

The unitary model is characterized by the assumption of constant Pareto weight μ. What is behind the assumption of constant Pareto weight? Solving the problem (1) is equivalent to solving the following Nash bargaining problem:

$$\max_{c_H, c_W, h_H, h_W, l_H, l_W} (U^H(c_H, l_H) - \bar{u}_H)^{\iota} (U^W(c_W, l_W) - \bar{u}_W)^{1-\iota}, \tag{2}$$
$$\text{s.t. } c_H + c_W \leq w_H h_H + w_W h_W, \quad 1 \geq l_H + h_H, \quad 1 \geq l_W + h_W,$$

where $\bar{u}_H$ and $\bar{u}_W$ are the values of the outside options of the bargaining (divorce) and $\iota \in (0, 1)$ is a constant. In addition, higher $\bar{u}_H$ and lower $\bar{u}_W$ imply larger μ. The proof of the equivalence is given by the Proposition A1 in the appendix A.

The critical point is that the assumption of the unitary model "μ is constant" holds if "$\bar{u}_H$ and $\bar{u}_W$ are constant." Therefore, the unitary model assumes that the values of the outside options of the bargaining are constant[7].

Next, consider another model:

[7]More precisely, the unitary model assumes that changes in the values of the outside options do not affect the allocation.

Definition 2 (Almås et al. (2023))**.** The collective model is a model under which a couple maximizes the sum of their private utilities with a flexible weight $\mu(\mathbf{z})$, for example,

$$\max_{c_H, c_W, h_H, h_W, l_H, l_W} \mu(\mathbf{z}) U^H(c_H, l_H) + (1 - \mu(\mathbf{z})) U^W(c_W, l_W), \tag{3}$$
$$\text{s.t. } c_H + c_W \leq w_H h_H + w_W h_W, \quad 1 \geq l_H + h_H, \quad 1 \geq l_W + h_W,$$

where $\mathbf{z}$ is a vector of some variables exogenous to the household.

This is called a collective model, which Chiappori (1988) and Chiappori (1992) have proposed. In contrast to the unitary model, the collective model allows the Pareto weight to vary. Similarly to the unitary model, applying Proposition A1, there exists a Nash bargaining problem behind the collective model (3):

$$\max_{c_H, c_W, h_H, h_W} (U^H(c_H, l_H) - \bar{u}_H(\mathbf{z}))^{\iota} (U^W(c_W, l_W) - \bar{u}_W(\mathbf{z}))^{1-\iota},$$
$$\text{s.t. } c_H + c_W \leq w_H h_H + w_W h_W, \quad 1 \geq l_H + h_H, \quad 1 \geq l_W + h_W.$$

Now, the values of the outside options of the bargaining are no longer constant but can depend on $\mathbf{z}$ since μ is not constant. This is the main difference between the unitary model and the collective model; the collective model allows the values of the outside options to vary while the unitary model assumes them to be constant[8].

Implications Consider the optimality conditions of the collective model (3). Suppose that U^i and Q are strictly increasing and concave in each argument and that c_i, l_i, and Q are gross substitutes. The first-order conditions imply

$$\mu(\mathbf{z}) U_c^H = (1 - \mu(\mathbf{z})) U_c^W, \tag{4}$$
$$\frac{U_l^H}{U_c^H} = w_H, \quad \frac{U_l^W}{U_c^W} = w_W, \tag{5}$$

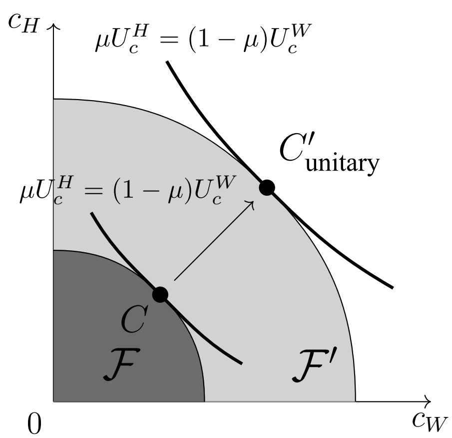
where subscripts of U^i denote partial derivatives.

Suppose the wage for wife w_W exogenously increases. In the unitary model, μ is constant. The rise in w_W increases c_W and decreases l_W by (5). Correspondingly, the increase in the wife's consumption spills over to the husband via (4); c_W also increases.

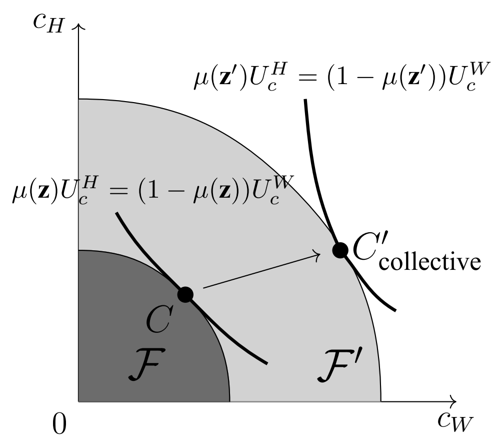
[8]More precisely, the collective model assumes that the values of the outside options can vary and affect the allocation.

What happens in the collective model is somewhat different. Suppose $\bar{u}_W(\mathbf{z})$ is increasing in w_W because the rise in w_W makes the wife better off after divorce. Then, as in the unitary model, the rise in w_W increases c_W and decreases l_W by (5). However, the spillover to the husband via (4) is offset by the decline in $\mu(\mathbf{z})$ because $\bar{u}_W(\mathbf{z})$ increases. As a result, the wife consumes a larger fraction of the increased income. This rise in the wife's bargaining power and resulting higher welfare is the bargaining effect, which can only be captured by a model with intra-household bargaining.

Figure 3 gives a graphical explanation. The x-axes and y-axes represent a wife's and a husband's consumption, respectively. The shaded areas represent the sets of consumptions that satisfy the first order conditions (5) and other constraints; $\mathcal{F}$ is that of before the wage increase and $\mathcal{F}'$ is after. The convex curves tangent to $\mathcal{F}$ and $\mathcal{F}'$ are the indifference curves in intra-household bargaining (equation (4)), and the points of contact, C, C'_{unitary}, and $C'_{\text{collective}}$, are the result of bargaining. In the unitary model (the left panel), the indifference curves are parallel, and both the husband and the wife equally benefit from the increase in the wife's wage. In contrast, in the collective model (the right panel), the indifference curves are not parallel; they shift, preferably toward the wife, due to the rise in her bargaining power. As a result, the wife's consumption increase is more significant than the husband's. The bargaining effect increases the welfare gain in favor of the wife.



(a) Unitary model



(b) Collective model

Figure 3: Comparison of Unitary and Collective models

Relation to Policy Evaluations How is this important? If intra-household bargaining is ignored, the policy evaluation is biased as much as the size of the bargaining effect. For example, there are several policies which promotes female labor participation, such as anti-gender-discrimination laws, the childcare subsidy, and Earned Income Tax Credit (EITC). These policies are expected to incentivize married women to work more. However, bargaining effect could offset the impacts of these policies. For example, if anti-gender-discrimination laws increase the female wage (higher w_W), it also increases the wife's bargaining power (higher $\bar{u}_W(\mathbf{z}) \to$ lower $\mu(\mathbf{z})$). Also, the childcare subsidy and EITC can help single mothers (higher $\bar{u}_W(\mathbf{z})$), which can reduce the wife's fear to divorce and increase the wife's bargaining power (lower $\mu(\mathbf{z})$). These increased bargaining power enables the wife to consume and take leisure more, which can offset the increase in their labor supply. Therefore, if the bargaining effect is ignored, the labor supply increase from these policies would be overestimated.

Income barriers, the main topic of this paper, are also subject to the bargaining effect. Without Income barriers, the labor supply by married women would be higher, which can reduce female wages because of excess supply of female labor relative to male labor. This wage decline can reduce welfare of single women, which leads to lower bargaining power. The lower bargaining power in turn reduces wife's consumption and leisure, which leads to higher labor supply. Therefore, if the bargaining effect is ignored, the labor supply increase from eliminating Income barriers would be underestimated.

Now, it is clear why intra-household bargaining matters. Most existing macroeconomic studies evaluate policies implicitly assuming a unitary model or treating a couple as a single individual. However, they ignore the bargaining effect, which can be substantially important. These impacts can be more prominent for policies that promote female labor participation. Empirical studies also support the collective model; several studies reject the unitary model while few reject the collective model[9]. To overcome these limitations, I suggest a dynamic model with explicit intra-household bargaining.

3.2 Extension to General Dynamic Setting

I extend the bargaining framework to a general dynamic setting which is applicable to wide range of models. Time t is discrete and finite, whose horizon is T. Let $s_t \in S$ ($|S| < \infty$) be an exogensous stochastic event in period t and $s^t \in S^t$ be the history of events up to t. I assume s_t follows the Markov process with $\pi(s^t)$ being the probability of s^t. Then, let $X_t \in \mathbb{R}^n$ be the state variable and $y_t \in \mathbb{R}^m$ be the control variable at time t. Also, define correspondences $\Gamma(X_t, s_t) : \mathbb{R}^n \times S \to \mathbb{R}^n$ as a convex compact set of the feasible state variables at time $t+1$ (X_{t+1}) given X_t and s_t, and $F(X_t, X_{t+1}, s_t) : \mathbb{R}^n \times \mathbb{R}^n \times S \to \mathbb{R}^m$ as a convex compact set of the feasible control variables at t (y_t) given X_t, X_{t+1}, and s_t. Denote by $U^i(y_t) : \mathbb{R}^m \to \mathbb{R}$ the instantaneous utility of $i \in \{H, W\}$

[9] Almås et al. (2023) summarize the literature. Lundberg et al. (1997) and Ward-Batts (2008) use the UK data and reject the unitary model by showing that the allowance to mothers and that to fathers exhibit different expenditure patterns. Attanasio and Lechene (2002) use Mexican data and similarly reject the unitary model. Regarding the collective model, Attanasio and Lechene (2014) tested it using Mexican data, but they do not reject the Pareto optimality of the resulting allocations. Cherchye et al. (2007), Cherchye et al. (2009), and Cherchye et al. (2011) test the collective model by revealed preference approach and do not reject it either.

at time t. Then, the set of Pareto optimal allocations is derived as follows:

Definition 3 (Set of Intra-Household Pareto Optimal Allocations). A set of allocations $(X_{t+1}(s^t), y_t(s^t))_{t=0}^T$ that satisfy the following conditions is the set of Pareto optimal allocations:

1. Given a sequence of $(\tilde{v}_t^H(s^t))_{t=1}^T$, $(X_{t+1}^W(s^t), y_t^W(s^t))_{t=1}^T$ solves

$$\begin{aligned}
&\max_{(X_{t+1}^W(s^t), y_t^W(s^t))_{t=1}^T} \sum_{t=1}^T \sum_{s^t \in S^t} \beta^t \pi(s^t) U^W(y_t^W(s^t)),\\
&\text{s.t.} X_{t+1}^W(s^t) \in \Gamma(X_t^W(s^t), s_t), \quad y_t^W(s^t) \in F(X_t^W(s^{t-1}), X_{t+1}^W(s^t), s_t), \quad \forall t,\\
&\sum_{\tau=t}^T \sum_{s^\tau \in S^\tau} \beta^{\tau-t} \pi(s^\tau|s^t) U^H(y_\tau^W(s^\tau)) \geq \tilde{v}_t^H(s^t), \quad \forall t\ \forall s^t \in S^t,\\
&X_1^W = X_1 \text{ given.}
\end{aligned}$$

2. Given a sequence of $(\tilde{v}_t^W(s^t))_{t=1}^T$, $(X_{t+1}^H(s^t), y_t^H(s^t))_{t=1}^T$ solves

$$\begin{aligned}
&\max_{(X_{t+1}^H(s^t), y_t^H(s^t))_{t=1}^T} \sum_{t=1}^T \sum_{s^t \in S^t} \beta^t \pi(s^t) U^H(y_t^H(s^t)),\\
&\text{s.t.} X_{t+1}^H(s^t) \in \Gamma(X_t^H(s^t), s_t), \quad y_t^H(s^t) \in F(X_t^H(s^{t-1}), X_{t+1}^H(s^t), s_t), \quad \forall t,\\
&\sum_{\tau=t}^T \sum_{s^\tau \in S^\tau} \beta^{\tau-t} \pi(s^\tau|s^t) U^W(y_\tau^H(s^\tau)) \geq \tilde{v}_t^W(s^t), \quad \forall t\ \forall s^t \in S^t,\\
&X_1^H = X_1 \text{ given.}
\end{aligned}$$

3. $(X_{t+1}^H(s^t), y_t^H(s^t))_{t=1}^T$ and $(X_{t+1}^W(s^t), y_t^W(s^t))_{t=1}^T$ are consistent, i.e.,

$$\begin{aligned}
y_t^H(s^t) &= y_t^W(s^t) =: y_t(s^t), && \forall t\ \forall s^t \in S^t,\\
X_{t+1}^H(s^t) &= X_{t+1}^W(s^t) =: X_{t+1}(s^t), && \forall t\ \forall s^t \in S^t.
\end{aligned}$$

These allocations are Pareto optimal in the sense that no other allocations can make the husband or the wife better off without making the other worse off. Correspondingly, the "bargaining rule" is defined as follows:

Definition 4 (Bargaining Rule). A bargaining rule describes relationship between $(\tilde{v}_t^H(s^t))_{t=1}^T$ and $(\tilde{v}_t^W(s^t)t)_{t=1}^T$, and pins down an equilibrium allocation $(X_{t+1}(s^t), y_t(s^t))_{t=0}^T$.

In static analog, in Figure 3, the set of Pareto optimal allocations is the boundary of $\mathcal{F}$, and the bargaining rule is the convex curve tangent to the boundary.

One example of the bargaining rule is the dynamic unitary model, which is often used in the family-macroeconomics literature.

Example 5 (Dynamic Unitary Model)**.** The bargaining rule of the dynamic unitary model with Pareto weight μ pins down the equilibrium allocation so that $(\tilde{v}_t^H(s^t))_{t=1}^T$ and $(\tilde{v}_t^W(s^t))_{t=1}^T$ solve,

$$\max_{X_{\tau+1}(s^\tau),y_\tau(s^\tau)} \sum_{\tau=t}^{T} \sum_{s^\tau \in S^\tau} \pi(s^\tau|s^t)\beta^{\tau-t}\left[\mu U^H(y_\tau(s^\tau)) + (1-\mu)U^W(y_\tau(s^\tau))\right],$$
$$\text{s.t. } X_{\tau+1}(s^\tau) \in \Gamma(X_\tau(s^{\tau-1}), s_\tau), \quad y_\tau(s^\tau) \in F(X_\tau(s^{\tau-1}), X_{\tau+1}(s^\tau), s_\tau), \quad \forall \tau,$$

for all t and $s^t \in S^t$.

Dynamic Nash Bargaining Model without Commitment In the next section, I construct a dynamic model which explicitly models the bargaining process. The value functions are defined as follows:

$$V_t^W(X_t, s^t) = \max_{(X_{\tau+1}^W(s^\tau),y_\tau^W(s^\tau))_{\tau=t}^T} \sum_{\tau=t}^{T} \sum_{s^\tau \in S^\tau} \beta^{\tau-t}\pi(s^\tau|s^t)U^W(y_\tau^W(s^\tau)),$$
$$\text{s.t.} X_{\tau+1}^W(s^\tau) \in \Gamma(X_\tau^W(s^\tau), s_\tau), \quad y_\tau^W(s^\tau) \in F(X_\tau^W(s^{\tau-1}), X_{\tau+1}^W(s^\tau), s_\tau), \quad \forall \tau,$$
$$\sum_{\tau=t'}^{T} \sum_{s^\tau \in S^\tau} \beta^{\tau-t'}\pi(s^\tau|s^{t'})U^H(y_\tau^W(s^\tau)) \geq \tilde{v}_{t'}^H(X_{t'}, s^{t'}), \quad \forall t' \, \forall s^{t'} \in S^{t'},$$

$$V_t^H(X_t, s^t) = \max_{(X_{\tau+1}^H(s^\tau),y_\tau^H(s^\tau))_{\tau=t}^T} \sum_{\tau=t}^{T} \sum_{s^\tau \in S^\tau} \beta^{\tau-t}\pi(s^\tau|s^t)U^H(y_\tau^H(s^\tau)),$$
$$\text{s.t.} X_{\tau+1}^H(s^\tau) \in \Gamma(X_\tau^H(s^\tau), s_\tau), \quad y_\tau^H(s^\tau) \in F(X_\tau^H(s^{\tau-1}), X_{\tau+1}^H(s^\tau), s_\tau), \quad \forall \tau,$$
$$\sum_{\tau=t'}^{T} \sum_{s^\tau \in S^\tau} \beta^{\tau-t'}\pi(s^\tau|s^{t'})U^W(y_\tau^H(s^\tau)) \geq \tilde{v}_{t'}^W(X_{t'}, s^{t'}), \quad \forall t' \, \forall s^{t'} \in S^{t'},$$

with $\tilde{v}_t^H(s^t)$ and $\tilde{v}_t^W(s^t)$ defined as

$$\tilde{v}_t^H(X_{t'}, s^t) = \sum_{\tau=t}^{T} \sum_{s^\tau \in S^\tau} \beta^{\tau-t} \pi(s^\tau | s^t) U^H(y_\tau^*(s^\tau)),$$
$$\tilde{v}_t^W(X_{t'}, s^t) = \sum_{\tau=t}^{T} \sum_{s^\tau \in S^\tau} \beta^{\tau-t} \pi(s^\tau | s^t) U^W(y_\tau^*(s^\tau)),$$

where

$$(X_{\tau+1}^*(s^\tau), y_\tau^*(s^\tau)) = \underset{X_{\tau+1}(s^\tau), y_\tau(s^\tau)}{\text{arg max}} \left[\begin{array}{c} \left(\sum_{\tau=t}^T \sum_{s^\tau \in S^\tau} \beta^{\tau-t} \pi(s^\tau|s^t) U^H(y_\tau(s^\tau)) - \bar{v}_t^H(X_t, s^t) \right)^\iota \\ \times \left(\sum_{\tau=t}^T \sum_{s^\tau \in S^\tau} \beta^{\tau-t} \pi(s^\tau|s^t) U^W(y_\tau(s^\tau)) - \bar{v}_t^W(X_t, s^t) \right)^{1-\iota} \end{array} \right],$$

$$\text{s.t.} X_{t+1}(s^t) \in \Gamma(X_t, s_t), \quad y_t(s^t) \in F(X_t, X_{t+1}(s^t), s_t), \quad \forall t.$$

where $\bar{v}_t^H(X_t, s^t)$ and $\bar{v}_t^W(X_t, s^t)$ are the values of outside options, i.e., the values of single men and women. Exploiting the Markov property, the recursive form of the dynamic Nash bargaining model without commitment is defined as follows:

Definition 6 (Dynamic Nash Bargaining Model without Commitment)**.** In the dynamic Nash bargaining model without commitment, given the values of outside options $(\bar{v}_t^H(X_t, s^t))_{t=1}^T$ and $(\bar{v}_t^W(X_t, s^t))_{t=1}^T$, a married couple solves

$$\begin{aligned} V_t^W(X, s) = \max_{X', y} U^W(y) + \beta \mathrm{E}[V_{t+1}^W(X', s') | X, s], \\ \text{s.t.} X' \in \Gamma(X, s), \quad y \in F(X, X', s), \quad V_t^H(X, s) \geq \tilde{v}_t^H(X, s), \\ V_{T+1}^W(X, s) = 0 \text{ given}, \end{aligned} \tag{6}$$

$$\begin{aligned} V_t^H(X, s) = \max_{X', y} U^H(y) + \beta \mathrm{E}[V_{t+1}^H(X', s') | X, s], \\ \text{s.t.} X' \in \Gamma(X, s), \quad y \in F(X, X', s), \quad V_t^W(X, s) \geq \tilde{v}_t^W(X, s), \\ V_{T+1}^H(X, s) = 0 \text{ given}, \end{aligned} \tag{7}$$

with $\tilde{v}_t^H(X,s)$ and $\tilde{v}_t^W(X,s)$ defined as

$$\tilde{v}_t^H(X,s) = U^H(y^*) + \beta \mathrm{E}[V_{t+1}^H(X'^*, s')|X,s],$$
$$\tilde{v}_t^W(X,s) = U^W(y^*) + \beta \mathrm{E}[V_{t+1}^W(X'^*, s')|X,s],$$

where

$$(X'^*, y^*) = \arg\max_{X',y} \begin{bmatrix} \left(U^H(y) + \beta \mathrm{E}[V_{t+1}^H(X',s')|X,s] - \bar{v}_t^H(X,s)\right)^{\iota} \\ \times \left(U^W(y) + \beta \mathrm{E}[V_{t+1}^W(X',s')|X,s] - \bar{v}_t^W(X,s)\right)^{1-\iota} \end{bmatrix}, \tag{8}$$
$$\text{s.t.} X' \in \Gamma(X,s), \quad y \in F(X,X',s).$$

The constraints (6) and (7) are terminal conditions. The concept of "without commitment" is based on Browning et al. (2014) and Mazzocco (2007). In this model, the bargaining (8) is held at every moment and every possible future event, which means no commitment is allowed. The absence of commitment is partially supported by the empirical evidence from Lise and Yamada (2019), which shows the commitment within the couple is limited and not perfect.

Compared to the often used unitary model (5), this model explicitly models the bargaining process and allows the values of the outside options to vary and affect the allocation, which endogenize bargaining powers and enables the capture of the *bargaining effect* as discussed in the previous subsection. Based on this theoretical framework, I construct a full dynamic model to evaluate Income barriers in the next section.

4 Model

4.1 Dynamic Decision Making of Household

Demographics There exists a continuum of households. A household consists of either one decision-maker (single) or two decision-makers (married). Each individual has a life-cycle with age denoted by $j \in \{1, 2, \dots, J\}$. They are born at age $j = 1$, retire at age $j = J_R$, and die by age $j = J$. They *ex-ante* differ in the sex and educational level. The sex is denoted by $s \in \{m, f\}$ and the educational level is denoted by $e \in \{c, nc\}$ (college and non-college graduates). Half of the population is men, and the other is women, while a fraction $\omega^e(s)$ of individuals are college graduates, and the rest are non-college graduates. The sex and the educational level do not change over time.

Marriage and Children Households also differ in marital and childcare status. The marital status is denoted by $\mathcal{M} \in \{M, S\}$ (married and single). A fraction $\omega^m(e)$ of individuals are born married, while the rest are born single and stay single throughout their lives. For simplicity, I assume that the spouses are of the same age. Also, the pair of educational levels between spouses is assortative, whose distribution is given by $F^m(e_H, e_W)$.

Married couples and single women are indexed by childcare status k; $k = 1$ if they have small kids and $k = 0$ otherwise. The "small kids" refers to kids so young that they need childcare (around 0-5 years old). Since they need childcare, small kids incurs additional costs for mothers to work outside, as described in the later paragraph. The law of motion of the childcare status is given by

$$\Pr(k' = 1 | k = 0) = \begin{cases} \pi^k(j) & \text{if married,} \\ 0 & \text{if single,} \end{cases} \tag{9}$$

$$\Pr(k' = 0 | k = 1) = \lambda^k(j). \tag{10}$$

A married couple without small kids experience childbirth and have small kids at the next age j with probability $\pi^k(j)$. If a household has small kids, the kids grow up with probability $\lambda^k(j)$, and the couple becomes without small kids. This model assumes that the children live with the mother if the couple divorces. Therefore, only divorced women can have small kids among single women. This assumption is based on the empirical fact that the number of single-mother households is much larger than that of single-father households.[10]

Income Dynamics The before-tax income of a worker is a product of the wage rate, labor efficiency, and the hours worked:

$$I = \underbrace{w_s}_{\text{wage rate}} \times \underbrace{\psi}_{\text{labor efficiency}} \times \underbrace{h}_{\text{hours worked}} . \tag{11}$$

The wage rate is common within the same sex s. The labor efficiency consists of

$$\psi = \begin{cases} \underbrace{\Psi(j, e)}_{\text{Deterministic trend}} \times \underbrace{\mathcal{E}}_{\text{Experience}} \times \underbrace{\exp(\nu + \eta)}_{\text{Idiosyncratic shock}} & \text{for women,} \\ \underbrace{\Psi(j, e)}_{\text{Deterministic trend}} & \text{for men.} \end{cases} \tag{12}$$

The first term represents the average life-cycle profile of wages, which is deterministic and depends on age and educational level. Since female workers are of my main interest, the productivity process for male workers is simpler and consists of only the deterministic trend.

[10] According to the Nationwide Survey on Single Parent Families in 2021 by MHLW, the number of single-mother households due to divorce is around 950 thousand, while that of single-father households is around 100 thousand.

The second term is also deterministic and represents skill depreciation while not working:

$$\log \mathcal{E}' = \log \mathcal{E} - \delta(e)\left(1 - \frac{h}{\bar{h}}\right),$$
$$\mathcal{E}_1 = 1, \tag{13}$$

where $\delta(e)$ is the depreciation rate which depends on the educational level e and $\bar{h}$ is the hours worked for full-time job. If a female worker works full-time, the labor efficiency does not depreciate. However, the labor efficiency depreciates if she works part-time or does not work. The skill depreciation based on the actual working hours is supported by Deming (2023), which suggests a causal relation between hours worked and wage growth.

The third term represents the idiosyncratic shock, which consists of permanent shock ν and transitory shock η. While the transitory shock is i.i.d. and follows the normal distribution, the permanent shock is persistent and follows the random walk process:

$$\begin{aligned} &\eta \sim N(0, \sigma_\eta^2), \\ &\nu' = \nu + \varepsilon, \quad \varepsilon \sim N(0, \sigma_\varepsilon^2), \quad \nu_1 = 0. \end{aligned} \tag{14}$$

This random walk process represents the empirical fact that the variance of log wage across workers linearly increases with age (Kitao and Yamada (2024), Lise et al. (2014)).

Tax and Social Securities The after-tax income is given by the functions below:

$$\begin{aligned} &\mathcal{I}^S(I, P, j) && \text{for single,} \\ &\mathcal{I}^M(I_H, I_W, P_H, P_W, j) && \text{for married,} \end{aligned}$$

where I is before-tax income. Since the income tax is determined jointly by the spouses' income, the tax function takes both the husband's and wife's income as arguments. Also, the tax function is age-dependent. Pension benefits are captured by this tax function.

P represents the benefits of the Employees' Pension. While the National Pension is universal, the amount of the Employees' Pension depends on the income level while working. Thus, P evolves over time:

$$P' = \begin{cases} \mathcal{P}(P, I, h) & \text{if } j < J_R, \\ P & \text{if } j \geq J_R. \end{cases} \tag{15}$$

The Employees' Pension benefits evolve depending on the income level and the hours worked while working. The functional form of $\mathcal{I}$ and $\mathcal{P}$ is specified in the section 5. In short, if a worker works longer and earns more, the pension benefits of the Employees' Pension increase.

Time Constraint Each individual allocates a unit of time to work, home production, and leisure. The time constraint is different between men and women and is given by

$$h^{\dagger} + q + l \leq 1, \quad \begin{cases} h = h^{\dagger} = \bar{h} & \text{if men,} \\ h = \max(h^{\dagger} - \kappa \mathbb{I}(k=1) - \zeta^{\mathcal{M}}, 0) & \text{if women.} \end{cases} \tag{16}$$

Recall that h is hours worked for a paid job in (11). In contrast, $h^{\dagger}$ is the hours worked including non-paid hours. For men, the hours worked are fixed to exogenous $\bar{h}$ (full-time hours)[11]. For women, the hours worked are not fixed, but there exists a set-up costs for working: $\zeta^{\mathcal{M}}$, which depends on marital status, and κ, which depends on childcare status. This formulation is motivated by the empirical fact that women's labor force participation rate declines after childbirth while that of men is almost constant and close to 1. The negative impacts of childbirth on female labor supplies are widely observed and especially large in Japan (Kleven et al. (2024)).

Preference The instantaneous utility function takes the form of CES:

$$U(c, Q, l) = \frac{\left[(\alpha_1 c^{1/\sigma} + \alpha_2 Q^{1/\sigma} + \alpha_3 l^{1/\sigma})^{\sigma}\right]^{1-\varsigma} - 1}{1 - \varsigma},$$

where c is the consumption, Q is the home production, l is the leisure, and $\alpha_1 + \alpha_2 + \alpha_3 = 1$. The production function of home production is given by

$$Q = \begin{cases} q^{\gamma_x} x^{1-\gamma_x}, & \text{if single,} \\ (q_H^{\gamma_q} + q_W^{\gamma_q})^{\gamma_x/\gamma_q} x^{1-\gamma_x}, & \text{if married,} \end{cases} \tag{17}$$

where q is the hours spent on home production, and x is the money spent on home production.

Budget Constraint Households face budget constraints:

$$\begin{cases} (1+\tau_c)(c+x) + a' \leq \mathcal{I}^S(I, P, j) + (1 + r(1-\tau_k))a & \text{if single,} \\ (1+\tau_c)(c_H + c_W + x) + a' \leq \mathcal{I}^M(I_H, I_W, P_H, P_W, j) + (1 + r(1-\tau_k))a & \text{if married,} \end{cases} \tag{18}$$

where a is the asset, a' is the asset in the next period, τ_k is the capital tax rate, and τ_c is the consumption tax rate. It is an important assumption that a couple perfectly shares the budget; they

[11] According to the Labour Force Survey, around 90% of men of working age have worked full-time in the last decades.

do not have separate accounts, and the income is pooled. Finally, households face the borrowing constraint,

$$a' \geq 0. \tag{19}$$

Recursive Formulation of Maximization Problem

Singles in Working Age ($j \in \{1, \ldots, J_R - 1\}$) The value functions of a single individual with sex s are given by

$$\begin{aligned} & v_S^{m,e}(j, a, P) \\ & = \max_{c,q,h^\dagger,l,x,a'} \{U(c, Q, l) + \beta \mathrm{E}[v_S^{m,e}(j+1, a', P')]\} && \text{if men,} \quad (20) \\ & v_S^{f,e}(j, a, \mathcal{E}, \eta, \nu, k, P) \\ & = \max_{c,q,h^\dagger,l,x,a'} \{U(c, Q, l) + \beta \mathrm{E}[v_S^{m,e}(j+1, a', \mathcal{E}', \eta', \nu', k', P')]\} && \text{if women,} \quad (21) \end{aligned}$$

subject to the constraints and law of motions (9), (10), (11), (12), (13), (14), (17), (15), (16), (18), and (19). Since a male worker supplies a fixed amount of hours $\bar{h}$ and his earnings are deterministic, his experience $\mathcal{E}$ does not depreciate. Also, if divorced, all children go with the mother. Therefore, $\mathcal{E}$ and k are not state variables of the value function for single men.

Singles in Retired Age ($j \in \{J_R, \ldots, J\}$) The value function of a single individual in retired age is given by

$$v_{s,e}^S(j, a, P) = \max_{c,q,l,x,a'} \big\{U(c, Q, l) + \pi^D(j)\beta \mathrm{E}[v_{s,e}^S(j+1, a', P')]\big\}, \tag{22}$$

subject to (17), (15), (18), (19), and time constraint $q + l \leq 1$. Since they no longer work, the labor efficiency and the permanent do not matter. $\pi^D(j)$ represents the survival probability of an individual at age j. Since the children become independent before retirement, the value function is not conditional on the number of children k.

Married Couples in Working Age ($j \in \{1, \ldots, J_R - 1\}$) A married couple solves the following maximization problem:

$$\max_{c_H, q_H, l_H, h_H^\dagger, c_W, q_W, l_W, h_W^\dagger, x, a'} \left[\begin{array}{l} \left(U(c_H, Q, l_H) + \beta \mathrm{E}[v_M^{m,\mathbf{e}}(j+1, a', \mathcal{E}', \eta', \nu', k', \mathbf{P}')] - (v_S^{m,e}(j, \theta_H a, P_H))\right)^\iota \\ \times\big(\underbrace{U(c_W, Q, l_W) + \beta \mathrm{E}[v_M^{f,\mathbf{e}}(j+1, a', \mathcal{E}', \eta', \nu', k', \mathbf{P}')]}_{\text{Objective function of Bellman equation}} - \underbrace{(v_S^{f,e}(j, \theta_W a, \mathcal{E}, \eta, \nu, k, P_W))}_{\text{Value of outside option}} \big)^{1-\iota} \end{array} \right] \tag{23}$$

subject to constraints (9), (10), (11), (12), (13), (14), (17), (16), (18), and (19). $\mathbf{P} = (P_H, P_W)$ is the vector of the Employees' Pension benefits of husband and wife.

The formulation is based on the discussion in section 3 The outside option of the bargaining is divorce, and its value is v_S, the value of a single individual. If divorced, the couple divides the assets, and the husband takes a fraction θ_H while the wife takes a fraction θ_W where $\theta_H + \theta_W = 1$.

The value functions are derived by substituting the optimal choices into the Bellman equation:

$$\begin{aligned} v_M^{m,\mathbf{e}}(j, a, \mathcal{E}, \eta, \nu, k, \mathbf{P}) &= U(c_H^*, Q^*, l_H^*) + \beta \mathrm{E}[v_M^{m,\mathbf{e}}(j+1, a'^*, \mathcal{E}', \eta', \nu', k', \mathbf{P}')], \\ v_M^{f,\mathbf{e}}(j, a, \mathcal{E}, \eta, \nu, k, \mathbf{P}) &= U(c_W^*, Q^*, l_W^*) + \beta \mathrm{E}[v_M^{f,\mathbf{e}}(j+1, a'^*, \mathcal{E}', \eta', \nu', k', \mathbf{P}')], \end{aligned}$$

where the variables with asterisks are the solution to the maximization problem (23).

If no feasible allocation makes both spouses better off than the single, the divorce occurs. Once divorced, the value functions are the same as the singles, but the children go with the mother:

$$\begin{aligned} v_M^{m,\mathbf{e}}(j, a, \mathcal{E}, \eta, \nu, k, \mathbf{P}) &= v_S^{m,e}(j, a, P_H), \\ v_M^{f,\mathbf{e}}(j, a, \mathcal{E}, \eta, \nu, k, \mathbf{P}) &= v_S^{f,e}(j, a, \mathcal{E}, \eta, \nu, k, P_W). \end{aligned}$$

Married Couples in Retired Age ($j \in \{J_R, \ldots, J\}$) A married couple in retired age solves the following maximization problem:

$$\max_{c_H, q_H, l_H, c_W, q_W, l_W, x, a'} \left[\begin{array}{l} \left(U(c_H, Q, l_H) + \pi^D(j) \beta \mathrm{E}[v_M^{m,\mathbf{e}}(j+1, a', \mathbf{P}')] - (v_S^{m,e}(j, \theta_H a, P_H))\right)^\iota \\ \times\big(U(c_W, Q, l_W) + \pi^D(j) \beta \mathrm{E}[v_M^{f,\mathbf{e}}(j+1, a', \mathbf{P}')] - (v_S^{f,e}(j, \theta_W a, P_W)) \big)^{1-\iota} \end{array} \right], \tag{24}$$

subject to constraints (17), (15), (18), (19), and time constraint $q + l \leq 1$. Then, the value functions are derived by substituting the optimal choices into the Bellman equation as in the working age.

4.2 Other Environment

Firm A representative firm produces the goods inputting the male and female labor with CES form:

$$Y = A(\pi_Y H_m^{\sigma_Y} + (1-\pi_Y)H_f^{\sigma_Y})^{1/\sigma_Y}, \tag{25}$$

where Y is the output, A is the total factor productivity, and H_m and H_f are the aggregated hours worked by male and female labor, respectively. The firm competitively maximizes the profit:

$$\max_{H_m, H_f} \{Y - w_m H_m - w_f H_f\}. \tag{26}$$

Government The government balances the budget each period:

$$G = \sum_{j=1}^{J} \tau_c C(j) + \sum_{j=1}^{J} \tau_k r K(j) + \sum_{j=1}^{J} \mathcal{I}^{Tax}(j), \tag{27}$$

where G is the government expenditure, $C(j)$ is the total consumption of final goods, $K(j)$ is total asset demands from households, and $\mathcal{I}^{Tax}$ is the income tax and pension contribution revenue from those aged j, respectively.

The definition of $C(j)$, $K(j)$, and $\mathcal{I}^{Tax}$, the market clearing conditions, and the definition of the stationary equilibrium are provided in the appendix B. The point is that only the labor market clears while other markets are not modeled; the interest rate r is exogenous.

5 Calibration

This section illustrates how the parameters are set in the model. The main dataset used is the Japanese Panel Survey of Consumers (JPSC), and others are taken from other datasets and literature. See appendix C for more details on the data sources and the calibration process.

5.1 Calibration Strategy

The calibration is conducted in two steps. First, I calibrate the parameters using only one cohort. I use cohort A from JPSC, who were born between 1958 and 1969, and assume every individual in the cohort was born in 1964. It is because this is the only cohort for which I have complete information on the life cycle. Over their life-cycles, the tax and social security systems have changed, which are modeled following the actual tax reforms.

Second, using the deep parameters calibrated in the first step, I construct a baseline economy with overlapping generations in a stationary equilibrium. For simplicity, I assume that the current tax and social security systems are fixed over time and cohorts are identical. The composition of the population across cohorts is exogenously given by data. Using the baseline economy, I simulate the counterfactual experiments.

In the model, I assume that the spouses are of the same age for simplicity, which is, however, not the case in the data. To match the data, I used the wife's age as the age of the couple, and the husband's age is set to be the same as the wife's.

5.2 Externally Calibrated Parameters

Parameter values are summarized in Table 2.

Table 2: Externally Calibrated Parameters

Param.	Description	Value	Source
Demographics			
J	Age of the oldest individual	35	92-93 years old
J_R	Age of retirement	19	60-61 years old
$\pi^D(j)$	Survival prob.	—	IPSS
$\pi^k(j)$	Prob. of having children	—	JPSC
$\lambda^k(j)$	Prob. of children growing up	—	JPSC
$\omega^e(m)$	Prob. of college grad. (males)	0.34	National Census
$\omega^e(f)$	Prob. of college grad. (females)	0.13	National Census
$F^m(e_H, e_W)$	Dist. of education of spouses	—	JPSC
$\omega^m(e)$	Prob. of marriage	0.83	National Census
$\mu(j)$	Population composition by age	—	Vital Statistics, IPSS
θ_H, θ_W	Assets division for husb. and wife	0.5, 0.5	
Preferences			
β	Subjective discount factor	0.96	
Productivity			
$\bar{h}$	Hours worked by full-time worker	0.37	JPSC
$\Psi(j, e)$	Wage profile	—	JPSC
σ_η	Std. dev. of transitory shocks	0.17	JPSC (Lise et al. (2014))
σ_ε	Std. dev. of permanent shocks	0.086	JPSC (Lise et al. (2014))
σ_Y	Elas. of sbstn. b/w male and female	0.45	Abbott et al. (2019)
Government			
τ_c	Consumption tax rate	—	3%-10%
τ^k	Capital income tax rate	0.20	20%
r	Interest rate	0.96%	BoJ

Note: The values for $\pi^D(j)$, $\pi^k(j)$, $\lambda^k(j)$, $F^m(e_H, e_W)$, $\mu(j)$, and $\Psi(j, e)$ are in Figure 4 and Figure A2.

Demographics One model period corresponds to two years. An individual enters the economy at age 24-25 and lives at most until age 92-93 ($J = 35$). S/he retires at age 60-61 ($J_R = 19$)[12], and the survival probability ($\pi^D(j)$) is based on the life table provided by the National Institute of Population and Social Security Research (IPSS). The probability of having children ($\pi^k(j)$) and that of child-growth ($\lambda^k(j)$) are computed from the JPSC data. Following Kitao and Mikoshiba (2022), I define a mother with children aged 0-5 as with small kids and others as without small kids. The specific values for $\pi^D(j)$, $\pi^k(j)$, $\lambda^k(j)$, $F^m(e_H, e_W)$, and $\mu(j)$ are shown in Figure A2 in appendix C.

Productivity The hours worked by full-time workers $\bar{h}$ is set to 0.371 (about 12 hours a weekday) based on the JPSC data. This is the average of the hours spent on work, study, and commuting by regular workers. The deterministic wage profile $\Psi(j, e)$ is presented in Figure 4 for each college and non-college graduate. This is computed by earnings profile of male regular workers, which I control by cohort dummies and the difference and the square of the difference between spouses' ages.

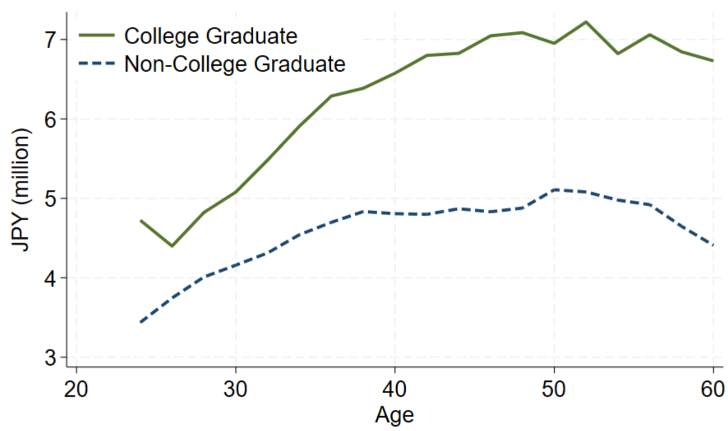


Figure 4: Wage Profile

Government The consumption tax rate τ_c is set to 3-10% based on the historical data, while the capital income tax rate τ^k is set to 20%.

The labor income tax consists of progressive national income tax and proportional local income tax. Regarding social security contributions (public pension + health insurance), there are three categories of pension insurance status. Category II insurees are employed workers whose working hours are sufficiently long.They have to pay the Employees' Pension contributions proportionally

[12] According to the General Survey on Working Conditions in 2022 by the Ministry of Health, Labour and Welfare, 94.4% of Japanese companies have a mandatory retirement age, 72.3% of which is 60 years old.

to income and are eligible for the Employees' Pension and the National pension benefits in the future. Category III insurees are the workers whose spouses are Category II insurees and whose income is below a certain threshold, JPY 1.3 million in 2024. They do not have to pay any public pension contribution but are eligible for the National pension benefits in the future. Category I insurees are the remainders, who have to pay the lump-sum the National pension contribution and are eligible for the National pension benefits in the future. A similar categorization is applied to health insurance. Such contributions and benefits are captured by the tax function $\mathcal{I}$ in the model.

In reality, the social security contributions are equally burdened by the employer and the employee, but I only consider the employee's side in the model for simplicity. This is based on the assumption that social security contributions do not affect employers' hiring behaviors or wage settings. This assumption can be interpreted as the labor market being perfectly competitive and employers not distinguishing workers by their social security status.

Households of retired age can receive pension benefits. All categories of pension insurees eligible for National pension benefits which are a fixed amount. In addition, category II insurees are eligible for the Employees' Pension benefits, which are proportional to the income level of the working age:

$$P = \sum_{j=1}^{J_R-1} \kappa^P(j) \times \min(I(j), \bar{I}(j)) \times \mathbb{I}_{\text{Cat II}}(j), \tag{28}$$

where P is the Employees' Pension benefits, $\kappa^P(j)$ is a certain coefficient, $I(j)$ is the before-tax income at age j, $\bar{I}(j)$ is an upper limit exogenously given by law, and $\mathbb{I}_{\text{Cat II}}(j)$ is the indicator function of whether the agent is category II insured or not at age j[13]. This evolution of the pension benefits is also captured by the function $\mathcal{P}$ in (15).

All precise values are shown in the appendix D.

5.3 Internally Calibrated Parameters

Other parameters are internally calibrated to match the model-generated moments with the empirical counterparts. The data moments consist of the gender wage gap by age for college graduates, that for non-college graduates, the labor force participation rate by age for married women, that for single women, and the consumption ratio between spouses by age for married couples. In this paper, I treat those actually working as the labor force participants and those who are not as non-participants regardless of their job search status. As in the wage profile, I control data moments for cohort dummies and the difference and the square of the difference between spouses' ages. The

[13] Precisely, the P evolves depending not on the annual income but on the monthly income.

calibrated parameter values are shown in Table 3 while the model and data moments are compared in Figure 5. TFP parameter A is set to normalize the wage rate for male workers w_m to one.

Table 3: Internally Calibrated Parameters

Param.	Description	value
κ_k	Cost for having kids	0.363
ζ^S	Set-up cost for market work (single women)	0.211
ζ^M	Set-up cost for market work (married women)	0.222
α_1	Coef. on consumption in utility	0.249
α_2	Coef. on home production in utility	0.300
σ	Intratemporal Elas. of sbstn. in utility	0.213
ς	Intertemporal Elas. of sbstn. in utility	2.441
γ_q	Elas. of sbstn. in husb. and wife's housework	0.599
γ_x	Coef. on home production function	0.643
δ_c	Dep. rate of skill for college grads	0.453
δ_{nc}	Dep. rate of skill for non-college grads	0.444
ι	Coeff. on Nash bargaining	0.398
π_Y	Weight on male labor in production	0.725
A	TFP	1.659

6 Numerical Analysis

6.1 Baseline Economy

Using the calibrated parameters, I construct a baseline economy. Figure 6 shows the baseline economy's income distributions for single and married women. The distribution of married women exhibits several bunchings while that of single women does not, which is a very similar pattern to that observed in tax data (Kondo and Fukai (2023))[14]. This fact suggests that this baseline economy depicts the distorted working behavior of married women because of Income barriers and is a good laboratory to study the effects of Income barriers.

6.2 Overview of Experiments

Using the baseline economy, this section conducts counterfactual experiments to eliminate Income barriers and quantify their impacts. The following experiments are conducted by comparing the baseline with counterfactual stationary equilibria:

[14]The bunchings in the model are larger than those in the data, which is because individuals in the model can choose their earnings more flexibly than in reality.

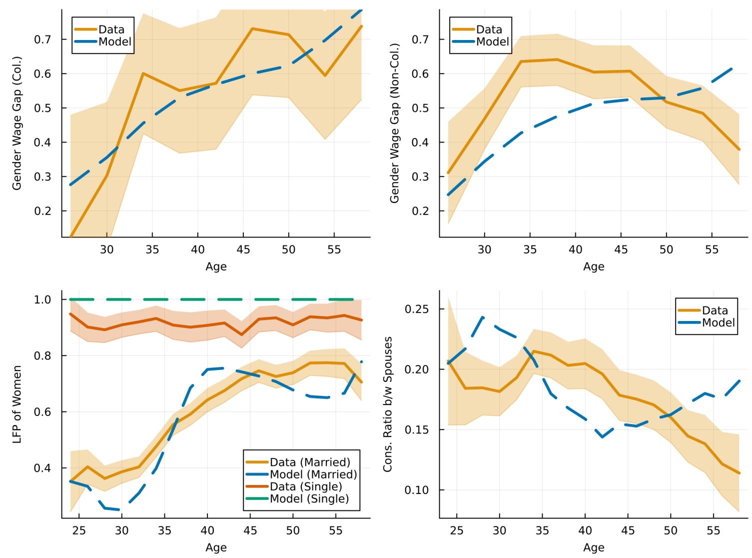


Figure 5: Model and Data Moments

Note: The shaded area shows the 95% confidence interval.

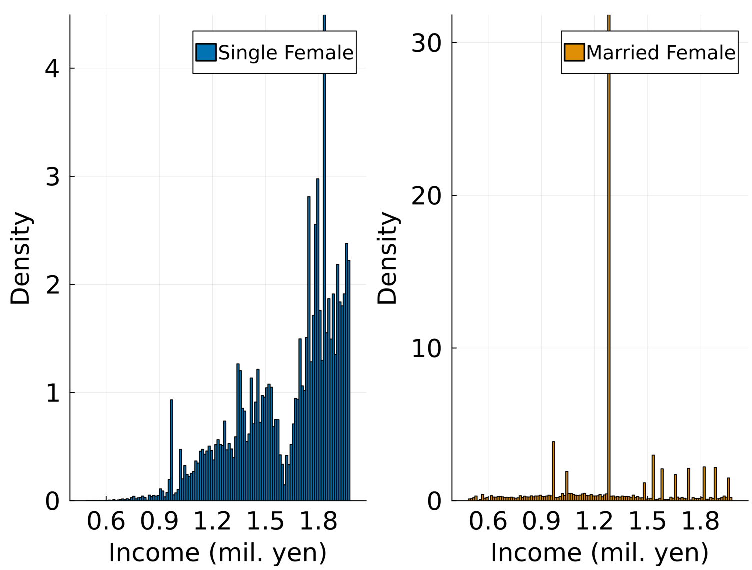


Figure 6: Income Distributions in the Baseline Economy

Note: The corresponding barriers to each bunching are conjectured as follows: The bunching at 1.0 million is due to a threashold of local income tax (Section D.2), that at 1.06 million is due to the Employees' Pension eligibility criteria (Section D.5), that at 1.3 million is the threshold for Category III of social security contributions (Section D.3), and those ranged from 1.5 to 2 million are due to spousal deductions (Section D.1).

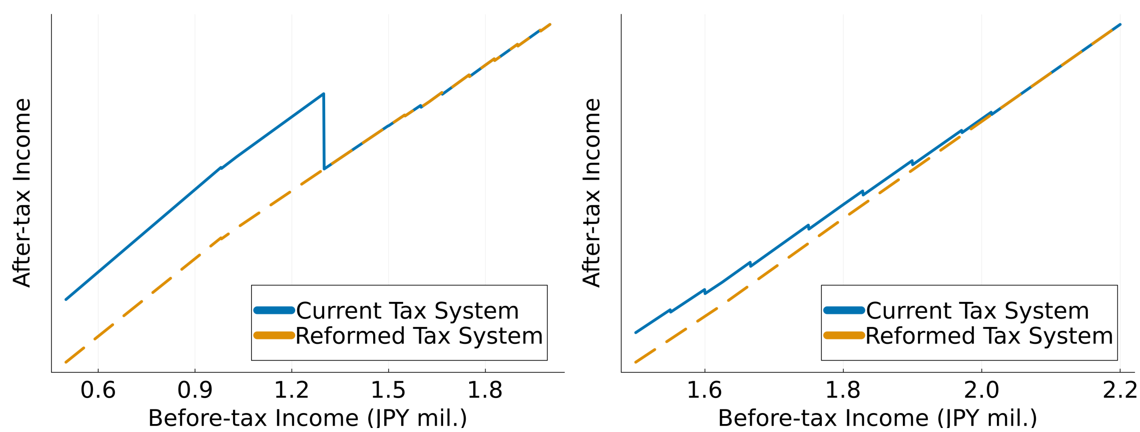
1. **1.3 mil. yen barrier**: Abolish the Category III status. Mandate all workers to pay pension and health insurance contributions as either Category I or II status.

2. **1.5 mil. yen barrier**: Abolish the spousal deductions[15].

3. **1.3 mil. yen barrier + 1.5 mil. yen barrier**: Combine the two policies above.

Experiments 1 to 3 focus on promoting the labor supply of wives[16]. Currently, if a wife earns less than JPY 1.3 million, she is exempted from paying pension and health insurance contributions while still being eligible for the benefits (Category III status). Experiment 1 removes this, and all workers are mandated to pay pension and health insurance contributions. Experiment 2 removes the spousal deductions. If a wife earns less than JPY 1.5 million, the husband can claim the spousal deductions and reduce the tax burden. As the wife's income increases above JPY 1.5 million, the spousal deductions gradually decrease, and the husband's tax burden increases; therefore, the marginal tax rate for wives is higher. Experiment 2 removes this barrier; although the overall tax burden increases, wives are not penalized for working more. Experiment 3 combines the two policies above.

Figure 7 graphically describes the each experiment. At this point, these policies are not revenue-neutral, assuming the government budget is balanced by adjusting the government expenditure. As a result, while Experiment 1 and 2 remove Income barriers, they increase the tax burdens.



(a) Experiment 1 **(b)** Experiment 2

Figure 7: Comparisons Between the Current and Counterfactual Tax Systems

Note: The x-axis is the before-tax income for the wife, and the y-axis is the sum of the after-tax incomes of the wife and husband. The husband's income is fixed at JPY 5 million. The solid line is the current system, and the dashed line is the reformed system in each experiment. Experiment 3 is the combination of Experiment 1 and 2.

[15] Both spousal deduction and spousal special deduction are abolished.

[16] As before, I call the secondary earner of a married couple "wife" and the primary earner "husband" for simplicity without any gender-discriminatory intention.

6.3 Results of Experiments

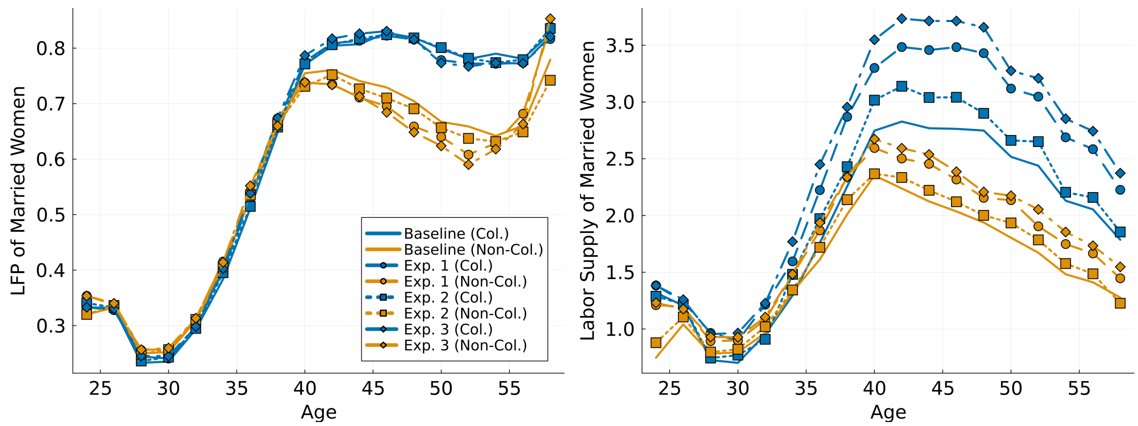
Labor Force Participation and Labor Supply Figure 9 shows the labor force participation rates and labor supply of married women, respectively for college and non-college graduates. First finding is the significant rise in the labor supply; the labor supply of married women increases by 20.4% in Experiment 3. As shown in the right panel, the increase is larger for college graduates than non-college graduates. This is because the wage is higher for college graduates and they are more likely to work until the earnings hit Income barriers. Another reason is that college graduate women are more likely to marry college graduate men, who earn more than non-college graduate men. This implies that college graduate husbands benefit from spousal deductions more, which makes 1.5 million yen barrier severer to wives. As a result, since college graduate wives are more likely to be stuck at the 1.5 million yen barrier, and the impacts of removing it is larger for them. Also, the rise in the labor supply is larger for Experiment 1 than for Experiment 2. This is natural because 1.3 million yen barrier is more severe than 1.5 million yen barrier, and more married women are subject to it as shown in Figure 6.

Second and paradoxical finding is the slight decline in the labor force participation rates, which is larger for non-college graduates. Since both policies remove barriers to female labor supply, the decline is puzzling. The answer comes from the equilibrium effect via labor market. By profit maximization of the firm, the increase in the relative labor supply of women to men leads to decline in the wage rate of women; while it increases by 1.9% for men, it decreases by 5.7% for women (Table 4). This decline discourages women from working, and for those whose earnings are so low and not stuck at Income barriers, this negative impact of wage decline exceeds the positive impact of removing the barriers, which leads to the decline in the labor force participation rates.

In sum, Income barriers have opposite effects on the intensive and extensive margins of married female labor supply. This feature can be captured only by models with market-based wage determination.

Aggregate Variables Table 4 shows the percentage changes in aggregate variables from the baseline. As discussed in the previous paragraph, the effects are opposite between intensive and extensive margins. The employment status also reflects this aspect; the while full-time workers increase, part-time workers decrease. The decrease in part-time workers is attributed to two directions. First, the removal of barriers let part-time workers work more and switch to full-time. Second, the resulted decline in wage rates discourages part-time workers from working and forces them out of the labor market.

The dynamics of other variables are quite natural. Since overall labor supply increases, aggregate consumption increases and leisure decreases. This increase also leads to increases in tax



(a) Labor Force Participation Rates (%)　　**(b)** Labor Supply

Figure 9: Labor Force Participation and Supply of Married Women

Note: Col. means college graduate, and Non-col. means non-college graduate. The legend is common to both figures.

revenues both in consumption and labor income tax. In addition, both experiments increase the tax burdens, which leads to increase in tax revenues, too. How government can redistribute this increase in tax revenues is an important question, which is discussed in the next subsection.

Table 4: Aggregate Variables (Change in % from the Baseline)

	Exp 1	**Exp 2**	**Exp 3**
Consumption	$+0.4$	$+0.1$	$+0.5$
Working Hours	$+3.1$	$+0.7$	$+3.8$
Leisure	-0.6	-0.2	-0.8
Labor Force Participation	-0.2	-0.4	-0.4
Labor Supply	$+3.0$	$+0.9$	$+3.7$
Full-Time	$+5.2$	$+2.3$	$+7.4$
Part-Time	-15.1	-7.9	-21.7
Income Tax Revenue	$+5.3$	$+1.8$	$+6.8$
Total Tax Revenue	$+9.1$	$+3.1$	$+11.7$
Wage Rate for Men	$+1.9$	$+0.6$	$+2.3$
Wage Rate for Women	-5.7	-1.8	-7.0

Note: Working hours are hours worked while labor supply is hours worked times labor efficiency. Tax revenues include consumption, labor income, and asset taxes and social security contributions.

Welfare Table 5 shows the change in the welfare measured by the consumption equivalent variation (CEV) from the baseline. The row "All" presents the Utilitarian welfare, the linear combination of the welfare weighted by the population. Since all three Experiments increase the tax burden, and the welfare for total population decreases. However, the effects are quite heterogeneous across groups. Basically, men are better off by removal of Income barriers, which is because the increase in wage rates (Table 4). Exceptionally, college graduate men are worse off in Experiment 2 and 3. Since these two experiments abolish spousal deductions, high-income husbands are penalized more by losing such deductions.

On the other hand, women significantly suffer from welfare loss. Although single women face no changes in tax system, they are worse off because of the decline in wage rates. Married women suffer more because in addition to the decline in wage rates, they face increased tax burden by removal of Category III status and spousal deductions. Also, the welfare loss is larger for non-college graduate married women. Since their wage is lower than college graduates, they are more likely to earn below barriers and benefit from Category III status and spousal deductions, which leads to larger welfare loss when they are removed.

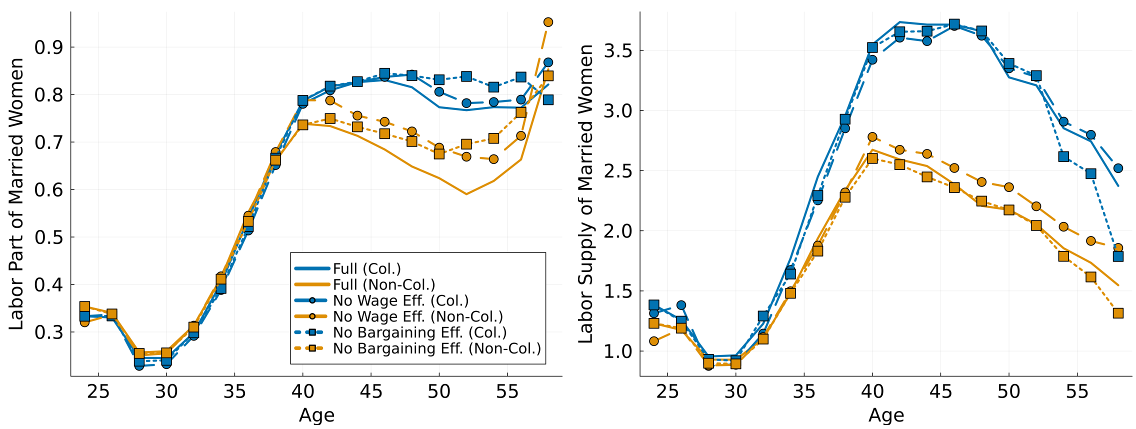
Table 5: Welfare (CEV in % from the Baseline)

	Exp 1	**Exp 2**	**Exp 3**
Men			
Single, Col.	$+1.1$	$+0.5$	$+1.5$
Single, Non-Col.	$+1.7$	$+0.5$	$+2.1$
Married, Col.	$+0.3$	-0.1	-0.5
Married, Non-Col.	$+2.8$	$+1.4$	$+1.0$
Women			
Single, Col.	-5.1	-1.6	-6.3
Single, Non-Col.	-5.4	-1.7	-6.6
Married, Col.	-6.0	-0.6	-8.3
Married, Non-Col.	-9.6	-4.7	-11.3
All	-5.8	-2.4	-7.1

Decomposition of Impacts: Intra-Household Bargaining and Labor Market By discussion so far, it is clear that the decline in female wage rates plays a crucial role in many dimensions, but how important is it? Also, as discussed in the previous sections, explicit incorporation of the intra-household bargaining is a key feature of this model, but how large is its effect? To answer these questions, taking Experiment 3 as an example, I decompose the impacts to isolate the effects via labor market and intra-household bargaining, respectively.

Figure 11 and Table 6 show the decomposition results. The column "Full" is the same as Experiment 3. The column "No Wage Eff." is the result without labor market effects; I fix the wage rate at the baseline level. The column "No Bargaining Eff." is the result without intra-household bargaining; I fix the bargaining power of spouses at the baseline level. The first key finding is the significant increase in labor supply without labor market effects. Wihout wage decline, the labor supply increase is larger, and the change in the labor force participation rate is positive while it is negative in the full model. This implies that the decline in female wage rates is crucial and even changes the sign of the labor force participation rate, which suggests the importance of modelling labor market.

The second and more insightful finding is the smaller increase in labor supply without *bargaining effect*. The increase in labor supply is 3.1% without bargaining effect. Since it is smaller than the full model, it leads to smaller decline in female wage rates, resulting the higher labor force participation rate. How does this happen? The removal of Income barriers increases female labor supply and declines female wage rates. This makes single women worse off, and since divorce is the outside option of the intra-household bargaining, the bargaining power of wives decreases. As a result, the share of consumption and leisure of wives within household decreases, which leads to the increase in the labor supply. In sum, the bargaining effect (higher labor supply – lower wage rate – lower bargaining power – higher labor supply) amplifies the labor supply increase. This is the most important insight of this paper – if intra-household bargaining is ignored, the results would be biased.



(a) Labor Force Participation Rates (%)

(b) Labor Supply

Figure 11: Decomposition of Labor Force Participation and Labor Supply of Married Women for Experiment 3 (%)

6.4 Revenue-Neutral Reform to EITC-like Tax System

In this subsection, I conduct another counterfactual experiment to seek for better tax system:

4. **Revenue-Neutral EITC**: Replace national income tax, local income tax, and social security contributions with revenue-neutral EITC-like income tax.

EITC is described later. As a counterfactual, I abolish national income tax, local income tax, and social security contributions (pension and health insurance). Then, I introduce EITC-like income

Table 6: Decomposition of Aggregate Variables for Experiment 3 (Change in % from the Baseline)

	No Wage Eff.	**No Bargaining Eff.**	**Full**
Consumption	$+0.8$	$+2.2$	$+0.5$
Working Hours	$+5.9$	$+3.4$	$+3.8$
Leisure	-1.1	-1.2	-0.8
Labor Force Part.	$+1.3$	$+0.9$	-0.4
Labor Supply	$+4.7$	$+3.1$	$+3.7$
Full-time Work	$+9.7$	$+4.2$	$+7.4$
Part-time Work	-21.5	-8.0	-21.7
Income Tax	$+8.0$	$+6.6$	$+6.8$
Total Tax	$+13.9$	$+12.6$	$+11.7$
Wage Rate for Men	–	$+1.5$	$+2.3$
Wage Rate for Women	–	-4.9	-7.0

tax, represented as

$$\mathcal{I}_{\text{EITC}}(I) = \tilde{\lambda} I^{1-\tilde{\tau}} \quad \text{for } j < J_R, \tag{29}$$

where I is the before-tax income, $\tilde{\lambda}$ is overall tax rate, and $\tilde{\tau}$ is the progressivity of the tax system. $\mathcal{I}_{\text{EITC}}$ is the after-tax income given before-tax income I. This specification is based on Heathcote et al. (2017). Since

$$\frac{\partial \mathcal{I}_{\text{EITC}}(I)}{\partial I} = (1-\tilde{\tau})\tilde{\lambda} I^{-\tilde{\tau}},$$

is decreasing in I, the income tax is progressive. In addition,

$$\frac{\mathcal{I}_{\text{EITC}}(I)}{I} = \tilde{\lambda} I^{-\tilde{\tau}} > 1 \quad \text{for } I < \tilde{\lambda}^{1/\tilde{\tau}}, \tag{30}$$

$$\mathcal{I}_{\text{EITC}}(0) = 0, \tag{31}$$

$$\frac{\partial \mathcal{I}_{\text{EITC}}(I)}{\partial I} \to \infty \text{ as } I \to 0. \tag{32}$$

EITC (Earned Income Tax Credit) is a tax system that provides both redistribution and labor supply incentives. Since EITC is a subsidy for low-income earners while being a tax for high-income earners (Equation (30)), it is redistributive. Besides, unlike the traditional welfare programs, EITC is available only for those working (Equation (31)). The marginal after-tax income is larger than one (Equation (32)), which means the wage rates that low-income earners face are higher, which incentivizes them to work more. EITC is widely used in OECD countries, including the U.S.[17], and

[17] EITC in the U.S. is available only for those with children.

causes no Income barriers. I seek revenue-neutral tax reform to eliminate Income barriers based on EITC.

A detailed explanation of this experiment is given below. In this experiment, I define the current net government revenue as

$$G = \text{Consumption tax} + \text{Capital tax} + \text{Income tax} + \text{Social security cont.} - \text{Pension benefits} \quad (33)$$

Then, I set the new tax function as (29) while keeping the consumption and capital tax unchanged. The formula for the amount of pension benefits is unchanged, while the financial source is changed from the social security contributions to the tax revenue. All workers are eligible for the Employees' Pension, whose benefits are calculated in the same way as the current system (Equeation (28)).

The progressive parameter $\tilde{\tau}$ is set to 0.181, an estimation from the U.S. data (Heathcote et al. (2017)). The level parameter $\tilde{\lambda}$ is calibrated internally so that G in (33) is the same as the current system (revenue-neutral). I compute the aggregate variables and CEVs based on the new tax system.

6.5 Results: Revenue-Neutral Tax Reform

Figure 12 compares the current and reformed system. Estimated revenue-neutral $\tilde{\lambda}$ is 1.02, represented as the dashed line. There is no Income barrier in the reformed system.

Figure 13 shows the labor force participation rates and labor supply for married women. The results of Experiment 3 are also shown for comparison. Now that the tax function is smooth and there are no barriers at all, which leads to larger labor supply increase. Also, unlike Experiment 3, the labor force participation rate increases. This is because the new tax system pays subsidies for low-income earners, which incentivizes them to work more. Table 7 shows the changes in aggregate variables. The tax revenue is neutral between the baseline and the new tax system. By redistributing the increased tax revenue to low-income earners as working subsidies, it realizes larger increase in the economy.

Table 8 shows the welfare changes. Unlike the previous experiments, the new tax system is welfare-improving for total population and beneficial for most groups. Since female labor supply increases, men can enjoy higher wage rates which compensate for the loss of spousal deductions. Also, since the new tax system pays subsidies for low-income earners, women, who are more likely to be low-income earners, are also better off by receiving the subsidies. Only exception is college graduate women, who are worse off. Since their wage is higher than non-college graduate women, they benefit less from EITC subsidies. Then, the increase in the labor supply non-college graduate women reduces the female wage rate, which leads to the welfare loss for college graduate women.

However, although they suffer from the welfare loss, its size is as half as that of Experiment 3. Therefore, this introduction of new EITC-like tax system is more effective than simply removing Income barriers.

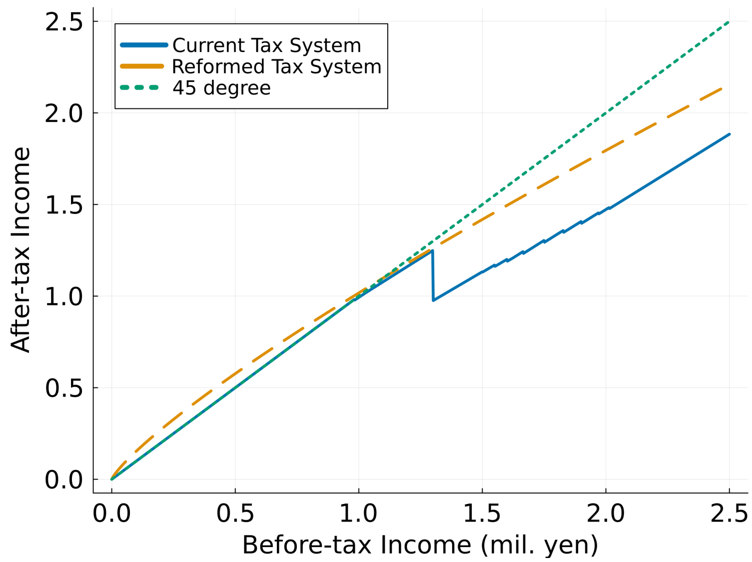
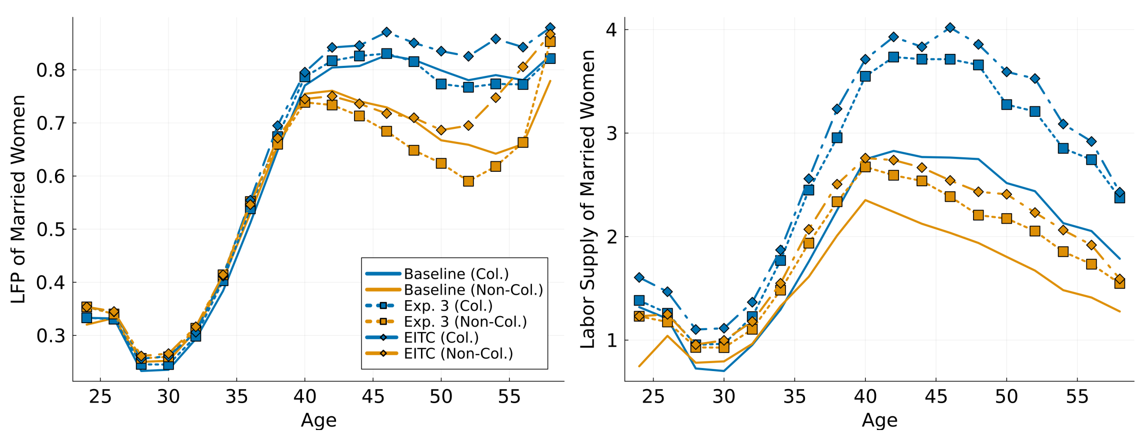


Figure 12: Comparison between the current and reformed system.

Note: The dashed line represents the result of revenue-neutral tax reform: $\tilde{\lambda} = 1.02$.



(a) Labor Force Participation Rates (%)

(b) Labor Supply

Figure 13: Labor Force Participation Rates and Labor Supply of Married Women (%)

7 Conclusion

This paper investigates the effects of Income barriers using a model with a dynamic intra-household bargaining structure. The analytical approach with simple models reveals the *bargaining effect* and

Table 7: Aggregate Variables (Change in % from the Baseline)

	Exp 3	**EITC**
Consumption	$+0.5$	$+2.4$
Working Hours	$+3.8$	$+4.9$
Leisure	-0.8	-1.1
Labor Force Participation	-0.4	$+1.6$
Labor Supply	$+3.7$	$+4.2$
Full-time	$+7.4$	$+7.8$
Part-time	-21.7	-15.5
Income Tax	$+6.8$	-0.9
Total Tax	$+11.7$	–
Wage Rate for Men	$+2.3$	$+2.5$
Wage Rate for Women	-7.0	-7.7

Table 8: Welfare (CEV in % from the Baseline)

	Exp 3	**EITC**
Men		
Single, Col.	$+1.5$	$+1.0$
Single, Non-Col.	$+2.1$	$+4.2$
Married, Col.	-0.5	$+2.3$
Married, Non-Col.	$+1.0$	$+6.1$
Women		
Single, Col.	-6.3	$+0.5$
Single, Non-Col.	-6.6	$+4.3$
Married, Col.	-8.3	-4.0
Married, Non-Col.	-11.3	$+0.9$
All	-7.1	$+2.1$

importance of considering intra-household bargaining in policy evaluations. Such effects are also observed in the quantitative model. The counterfactual experiments with a model calibrated by JPSC data show that removing Income barriers increases the labor supply of married women by 20.4%. However, due to the female wage decline caused by this increase in the labor supply, the effect on extensive margin is opposite to intensive margin; the labor force participation rate declines. Also, if the intra-household bargaining is ignored, the increase in the labor supply is underestimated. From the view point of welfare, while simple elimination of Income barriers can cause welfare loss, introduction of revenue-neutral EITC can cause 2.1% of welfare gain. The main contribution of this paper would be two-fold: the suggestion of considering intra-household bargaining in policy evaluations and the suggestion of alternative policy to eliminate Income barriers.

I end this paper with three remarks. First, the households are abstracted in this paper. This paper mainly focuses on working married women. Children, who face the 1.03 million yen barrier and are often subject to policy discussions, are not considered in this paper. Male workers are also very simple, whose productivity is almost homogeneous, and labor supply is inelastic. Also, this paper ignores the labor supply of or taxation on older people, who are also subject to the policy discussions. Second, the firm side is too abstract in this paper. For example, Japanese private firms often pay spousal allowance to married employees if their spouses' earnings are low. This can affect married female working behavior, but since this paper focuses on the policy, this is not considered. Also, in reality, social security contributions are burdened equally by employers and employees. Who pays the contributions does not matter in a perfectly competitive labor market, but if imperfect competition is considered, it matters, and the results can be different. Third, in reality, the commitment within a couple is partially possible. Lise and Yamada (2019) find that small shocks to wage can be insured between spouses while large shocks cannot. Besides, the calibration result (Figure 5) shows a more minor change in relative consumption in data than in the model, suggesting that the commitment is partially possible in reality. Therefore, the assumption of no commitment in this paper can be too strong, and another model with partial commitment can be considered. This point is left for future research.

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A Mathematical Appendix

A.1 The Equivalence between the Nash Bargaining Solution and the Unitary Model (Section 3)

In the main text, I mentioned the equivalence of solving (1) and (2). As a proof, I provide the following generalized proposition. Browning et al. (2014) is referred to for the proof.

Proposition A1. *Let $U^i(\cdot,\cdot) : \mathbb{R}^n \times \mathbb{R}^m \rightarrow \mathbb{R}$ be a utility function of $i \in \{H, W\}$, which is continuous, strictly increasing and strictly concave.*

Fix exogenous parameter $\iota \in (0,1)$. Given values of outside options $\bar{u}_H$ and $\bar{u}_W$, suppose $(\mathbf{x}^{H*}, \mathbf{x}^{W*}, \mathbf{X}^*)$ is the solution to the following problem:

$$\max_{\mathbf{x}^H, \mathbf{x}^W, \mathbf{X}} (U^H(\mathbf{x}^H, \mathbf{X}) - \bar{u}_H)^{\iota} (U^W(\mathbf{x}^W, \mathbf{X}) - \bar{u}_W)^{1-\iota}, \quad (34)$$
$$s.t. \quad \mathbf{p}'(\mathbf{x}^H + \mathbf{x}^W) + \mathbf{P}'\mathbf{X} \leq I,$$

where $\mathbf{p} \in \mathbb{R}^n_+$ and $\mathbf{P} \in \mathbb{R}^m_+$ are price vectors and $I \in \mathbb{R}_+$ is the total income. Then, there exists a constant μ under which $(\mathbf{x}^{H*}, \mathbf{x}^{W*}, \mathbf{X}^*)$ is a solution to the following problem:

$$\max_{\mathbf{x}^H, \mathbf{x}^W, \mathbf{X}} \mu U^H(\mathbf{x}^H, \mathbf{X}) + (1-\mu) U^W(\mathbf{x}^W, \mathbf{X}), \quad (35)$$
$$s.t. \quad \mathbf{p}'(\mathbf{x}^H + \mathbf{x}^W) + \mathbf{P}'\mathbf{X} \leq I.$$

The converse is also true; if $(\mathbf{x}^{H*}, \mathbf{x}^{W*}, \mathbf{X}^*)$ is a solution to (35)*, then there exists a constant μ under which $(\mathbf{x}^{H*}, \mathbf{x}^{W*}, \mathbf{X}^*)$ is a solution to* (34)*.*

In addition, larger $\bar{u}_H$ and smaller $\bar{u}_W$ imply larger μ.

In this context, $\mathbf{x}^H$ and $\mathbf{x}^W$ are the private consumptions while $\mathbf{X}$ is the public consumption.

Proof. Let $\mathcal{S} \subset \mathbb{R}^2$ be the set of feasible utility pairs (U^H, U^W) under the budget constraint. First, show that $\mathcal{S}$ is strictly convex. For any $(U^H, U^W), (U^{H'}, U^{W'}) \in \mathcal{S}$ and corresponding consumptions $(\mathbf{x}^H, \mathbf{x}^W), (\mathbf{x}^{H'}, \mathbf{x}^{W'})$, take any $\lambda \in (0,1)$. Then, define

$$\mathbf{x}^{H''} = \lambda \mathbf{x}^H + (1-\lambda)\mathbf{x}^{H'},$$
$$\mathbf{x}^{W''} = \lambda \mathbf{x}^W + (1-\lambda)\mathbf{x}^{W'},$$
$$\mathbf{X}'' = \lambda \mathbf{X} + (1-\lambda)\mathbf{X}'.$$

Since $(U^H, U^W), (U^{H'}, U^{W'}) \in \mathcal{S}$,

$$\begin{aligned}
&\mathbf{p}'(\mathbf{x}^{H''} + \mathbf{x}^{W''}) + \mathbf{P}'\mathbf{X}'' \\
=&\lambda \mathbf{p}'(\mathbf{x}^{H} + \mathbf{x}^{W}) + \mathbf{P}'\mathbf{X} + (1-\lambda)\mathbf{p}'(\mathbf{x}^{H'} + \mathbf{x}^{W'}) + \mathbf{P}'\mathbf{X}' \\
\leq&\lambda I + (1-\lambda) I = I,
\end{aligned}$$

which implies $\mathbf{x}^{H''}, \mathbf{x}^{W''}, \mathbf{X}''$ are feasible, and $(U^H(\mathbf{x}^{H''}, \mathbf{X}''), U^W(\mathbf{x}^{W''}, \mathbf{X}'')) \in \mathcal{S}$. Since $U^i(\cdot,\cdot)$ is strictly concave,

$$\begin{aligned}
U^H(\mathbf{x}^{H''}, \mathbf{X}'') > \lambda U^H(\mathbf{x}^{H}, \mathbf{X}) + (1-\lambda) U^H(\mathbf{x}^{H'}, \mathbf{X}'), \\
U^W(\mathbf{x}^{W''}, \mathbf{X}'') > \lambda U^W(\mathbf{x}^{W}, \mathbf{X}) + (1-\lambda) U^W(\mathbf{x}^{W'}, \mathbf{X}').
\end{aligned}$$

Since $U^i(\cdot,\cdot)$ is contnuous and strictly increasing, there exists a feasible consumption which coincides with a utility pair $(\tilde{U}^H, \tilde{U}^W) := (\lambda U^H(\mathbf{x}^H, \mathbf{X}) + (1-\lambda) U^H(\mathbf{x}^{H'}, \mathbf{X}'), \lambda U^W(\mathbf{x}^W, \mathbf{X}) + (1-\lambda) U^W(\mathbf{x}^{W'}, \mathbf{X}'))$. Thus, $(\tilde{U}^H, \tilde{U}^W) \in \mathcal{S}$ and $\mathcal{S}$ is convex. In addition, since $U^i(\mathbf{x}^{i''}, \mathbf{X}) > \tilde{U}^i$ for $i \in \{H, W\}$ and $(U^H(\mathbf{x}^{\mathbf{H}''}, \mathbf{X}), U^W(\mathbf{x}^{\mathbf{W}''}, \mathbf{X})) \in \mathcal{S}$, $(\tilde{U}^H, \tilde{U}^W)$ is an interior point of $\mathcal{S}$[18]. Therefore, $\mathcal{S}$ is strictly convex.

Then, the problems (34) and (35) are rewritten as

$$\max_{(U^H, U^W) \in \mathcal{S}} W^N(U^H, U^W) := (U^H - \bar{u}_H)^{\iota} (U^W - \bar{u}_W)^{1-\iota}, \tag{36}$$

$$\max_{(U^H, U^W) \in \mathcal{S}} W^U(U^H, U^W) := \mu U^H + (1-\mu) U^W. \tag{37}$$

The level curves of $W^N, W^U : \mathbb{R}^2 \to \mathbb{R}$ are concave to the origin. Then, the solutions to (36) and (37) are the tangency points of the level curves of W^N and W^U with the boundary of $\mathcal{S}$[19]. The

[18]Note that a consumption profile which returns a slightly smaller utility than $(\tilde{U}^H, \tilde{U}^W)$ is obviously feasible because of the continuity and strict increasingness of $U^i(\cdot,\cdot)$.

[19]The existence of the solution (the tangency point) is guaranteed because the objective functions of (34) and (35) are continuous and constraints are compact, which implies the existence of the maximum for (34) and (35). The uniqueness is guaranteed by the concavity of the level curves and the strict convexity of $\mathcal{S}$.

slope of the tangent line to the level curves of W^N and W^U at (U^H, U^W) is given by

$$\begin{aligned} dW^N &= \frac{\partial W^N}{\partial U^H} dU^H + \frac{\partial W^N}{\partial U^W} dU^W, \\ &= \iota \left(\frac{U^W - \bar{u}_W}{U^H - \bar{u}_H} \right)^{1-\iota} dU^H + (1-\iota) \left(\frac{U^H - \bar{u}_H}{U^W - \bar{u}_W} \right)^{\iota} dU^W, \\ dW^U &= \frac{\partial W^U}{\partial U^H} dU^H + \frac{\partial W^U}{\partial U^W} dU^W, \\ &= \mu dU^H - (1-\mu) dU^W, \end{aligned}$$

which implies

$$-\frac{dU^H}{dU^W} = \begin{cases} \frac{\iota}{1-\iota} \left(\frac{U^W - \bar{u}_W}{U^H - \bar{u}_H} \right) & \text{for the level curves of } W^N, \\ \frac{\mu}{1-\mu} & \text{for the level curves of } W^U. \end{cases}$$

Then, if

$$\frac{\iota}{1-\iota} \left(\frac{U^W - \bar{u}_W}{U^H - \bar{u}_H} \right) = \frac{\mu}{1-\mu}, \tag{38}$$

holds, (U^H, U^W) is the tangency point of the both level curves of W^N and W^U.

Finally, show the equivalence of the solutions. Fix exogenous parameter $\iota \in (0, 1)$. Take any Pareto weight $\mu \in (0, 1)$, and let $(\mathbf{x}^{H*}, \mathbf{x}^{W*}, \mathbf{X}^*)$ be the solution to (34) and (U^{H*}, U^{W*}) be the corresponding utility pair. Then, by choosing $(\bar{u}_H, \bar{u}_W)$ so that (38) holds, $(\mathbf{x}^{H*}, \mathbf{x}^{W*}, \mathbf{X}^*)$ is the solution to (35) too. The converse is also true. Take any value of outside options $(\bar{u}_H, \bar{u}_W)$, and let $(\mathbf{x}^{H*}, \mathbf{x}^{W*}, \mathbf{X}^*)$ be the solution to (35) and (U^{H*}, U^{W*}) be the corresponding utility pair. Then, by choosing μ so that (38) holds, $(\mathbf{x}^{H*}, \mathbf{x}^{W*}, \mathbf{X}^*)$ is the solution to (34) too.

Additionaly, (38) requires that larger $\bar{u}_H$ and smaller $\bar{u}_W$ imply larger μ to satisfy the equivalence. Now, the proposition is proven. □

Figure A1 illustrates the equivalence between the Nash bargaining solution and the unitary model. The shaded area represents the set $\mathcal{S}$ of feasible utility pairs under the budget constraint. The line and concave curve represent the level curves of the unitary model (W^U) and the Nash bargaining solution (W^N), respectively. By appropriately choosing μ or $(\bar{u}_H, \bar{u}_W)$, I can let the level curves be tangent to $\mathcal{S}$ at the same point, which implies the equivalence of the solutions.

B Definition of Stationary Equilibrium

Based on the model in section 4, I define the stationary equilibrium as follows. The stationary equilibrium consists of

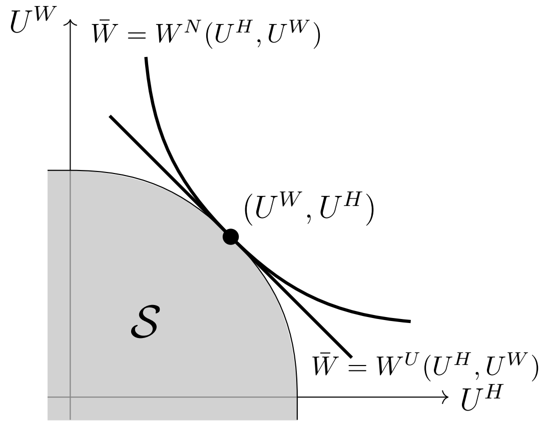


Figure A1: Comparison of Unitary and Collective models

1. Value functions for single households $v_S^{s,e}$ ($s \in \{m, f\}$ denotes the sex and $e \in \{c, nc\}$ denotes the education level (college graduate or not)) and for married households $v_M^{s,\mathbf{e}}$ ($s \in \{m, f\}$, and $\mathbf{e} = (e_H, e_W) \in \{c, nc\}^2$ denotes the pair of education levels of the husband and the wife).

2. Policy functions of consumption, working hours, leisure, housework, expenditure to home production, and savings for single households $(c_S^{s,e}, (h^\dagger)_S^{s,e}, l_S^{s,e}, q_S^{s,e}, x_S^{s,e}, (a')_S^{s,e})$ ($s \in \{m, f\}$ and $e \in \{c, nc\}$) and for married households $(c_M^{s,\mathbf{e}}, (h^\dagger)_M^{s,\mathbf{e}}, l_M^{s,\mathbf{e}}, q_M^{s,\mathbf{e}}, x_M^{s,\mathbf{e}}, (a')_M^{s,\mathbf{e}})$ ($s \in \{m, f\}$ and $\mathbf{e} = (e_H, e_W) \in \{c, nc\}^2$).

3. Representative firm's labor demand and production: H_m, H_f, Y.

4. Government expenditure: G.

5. Prices: w_m, w_f.

6. Distribution of the single households $\Phi_S^{s,e}$ ($s \in \{m, f\}$ and $e \in \{c, nc\}$) and the married households $\Phi_M^{s,\mathbf{e}}$ ($s \in \{m, f\}$ and $\mathbf{e} = (e_H, e_W) \in \{c, nc\}^2$).

In the stationary equilibrium, given the exogenous interest rate r, the following conditions hold.

1. Given the prices w_m, w_f, the policy functions solve the optimization problems of the households ((20), (21), (22), (23), and (24)) given constraints ((9) - (19)), where the value functions are associated with the solutions.

2. Given the prices w_m, w_f, the representative firm's labor demand and production solve the optimization problem (26) with the production function (25).

3. Labor market clears competitively:

$$H_m = \sum_{j=1}^{J_R-1}\left(\sum_{e\in\{c,nc\}}\int\left[\Psi(e,j)\bar{h}\right]d\Phi_S^{m,e} + \sum_{\mathbf{e}\in\{c,nc\}^2}\int\left[\Psi(e_H,j)\bar{h}\right]d\Phi_M^{m,\mathbf{e}}\right),$$

$$H_f = \sum_{j=1}^{J_R-1}\left(\sum_{e\in\{c,nc\}}\int\left[\Psi(e,j)\mathcal{E}\exp(\nu+\eta)h_S^{f,e}\right]d\Phi_S^{f,e}\right.$$
$$\left. + \sum_{\mathbf{e}\in\{c,nc\}^2}\int\left[\Psi(e_W,j)\mathcal{E}\exp(\nu+\eta)h_M^{f,\mathbf{e}}\right]d\Phi_M^{f,\mathbf{e}}\right),$$

where $h = \max(h^\dagger - \kappa\mathbb{I}(k=1), 0)$ is the optimal hours worked. Arguments of the distribution functions are omitted for brevity.

4. G balances the government budget constraint (27), where

$$C(j) := \sum_{s\in\{m,f\}}\left(\sum_{e\in\{c,nc\}}\int(c_S^{s,e}+x_S^{s,e})d\Phi_S^{s,e} + \sum_{\mathbf{e}\in\{c,nc\}^2}\int(c_M^{s,\mathbf{e}}+x_M^{s,\mathbf{e}})d\Phi_M^{s,\mathbf{e}}\right),$$

$$K(j) := \sum_{s\in\{m,f\}}\left(\sum_{e\in\{c,nc\}}\int a\,d\Phi_S^{s,e} + \sum_{\mathbf{e}\in\{c,nc\}^2}\int a\,d\Phi_M^{s,\mathbf{e}}\right),$$

$$\mathcal{I}^{Tax}(j) := \sum_{s\in\{m,f\}}\sum_{e\in\{c,nc\}}\left(\int\left[\underbrace{I_S^{s,e} - \mathcal{I}^S(I_S^{s,e},P,j)}_{\text{Tax revenue from single households}}\right]d\Phi_S^{s,e}\right)$$
$$\sum_{\mathbf{e}\in\{c,nc\}^2}\left(\int\frac{1}{2}\left[\underbrace{I_M^{m,\mathbf{e}} + I_M^{f,\mathbf{e}} - \mathcal{I}^M(I_M^{m,\mathbf{e}}, I_M^{f,\mathbf{e}}, \mathbf{P}, j)}_{\text{Tax revenue from married households}}\right]d\Phi_M^{s,\mathbf{e}}\right), \tag{39}$$

where, for $j < J_R$,

$$I_S^{m,e}(j,a,P) = w_m\Psi(e,j)\bar{h}$$
$$I_S^{f,e}(j,a,\mathcal{E},\nu,\eta,k,P) = w_f\Psi(e,j)\mathcal{E}\exp(\nu+\eta)h_S^{f,e}(a,\mathcal{E},\nu,\eta,k,P)$$
$$I_M^{m,\mathbf{e}}(j,a,\mathcal{E},\nu,\eta,k,\mathbf{P}) = w_m\Psi(e_H,j)\bar{h}$$
$$I_M^{f,\mathbf{e}}(j,a,\mathcal{E},\nu,\eta,k,\mathbf{P}) = w_f\Psi(e_W,j)\mathcal{E}\exp(\nu+\eta)h_M^{f,e}(a,\mathcal{E},\nu,\eta,k,\mathbf{P}),$$

and for $j \geq J_R$,

$$I_S^{m,e}(j,a,P) = I_S^{f,e}(j,a,\mathcal{E},\nu,\eta,k,P) = I_M^{m,e}(j,a,\mathcal{E},\nu,\eta,k,\mathbf{P}) = I_M^{f,e}(j,a,\mathcal{E},\nu,\eta,k,\mathbf{P}) = 0.$$

Arguments of the distribution functions are omitted for brevity[20].

5. The initial distribution of the household is externally given (section 5[21]). After the initial distribution, the distribution evolves according to the policy functions of assets and lows of motions for $\mathcal{E}$, ν, η, k, and P. The total population by age is normalized to exogenously given $\mu(j)$ for all age j, where $\sum_j \mu(j) = 1$.

C Data Appendix

The main dataset for this analysis, the Japan Panel Survey of Consumers (JPSC), was provided by the Keio University Panel Data Research Center[22].

C.1 Demographics

For the survival probability by age $\pi^D(j)$, I use the life expectancy table of the National Institute of Population and Social Security Research (IPSS) and exploit the latest cross-sectional data in 2022. The population composition by age $\mu(j)$ is based on the annual growth rate of the number of births (-1.44%) computed from Vital Statistics. For those aged above 60, I adjust the numbers by the survival probability and normalize the total population measure to 1. The probability of college enrollment by age $\omega^e(m), \omega^e(f)$ (for men and women) is obtained by the National Census, while Assortativeness of marriage, or the distribution of the spouse's educational level $F^m(e_H, e_W)$, is obtained from JPSC.

The Poisson rate of childbirth $\pi^k(j)$ and that of child-growth $\lambda^k(j)$ are obtained as follows. First, I calculate the fraction of couples with small kids (aged 0-5) for each wife's age. Second, I calculate the probability of spouses with small kids having no kids in the next period, which is the Poisson rate of childbirth $\lambda^k(j)$. Finally, I obtain the Poisson rate of childbirth $\pi^k(j)$ to match the fraction of couples with small kids in the data given $\lambda^k(j)$. I set $\theta_H = \theta_W = 0.5$, assuming that the husband and wife equally share their assets if divorced based on the principle.

C.2 Productivity

To obtain the wage profile $\Psi(j, e)$, using JPSC, I conduct the following regression for each college and non-college graduate separately:

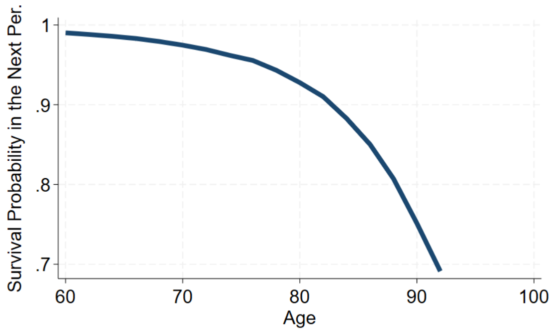
$$Y_{ijc} = \boldsymbol{\beta}'_{\text{age}} \boldsymbol{D}_{\text{age}} + \boldsymbol{\beta}'_{\text{cohort}} \boldsymbol{D}_{\text{cohort}} + \boldsymbol{\delta}' \boldsymbol{X}_{ijc} + \varepsilon_{ijc},$$

[20] In (39), the tax revenue is multiplied by $1/2$ because it is the total tax revenue from a couple but counted twice for the husband and the wife.

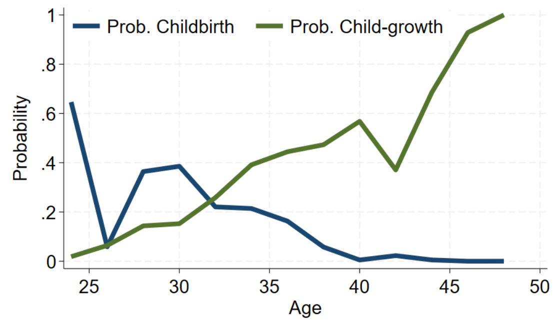
[21] Half of the households are men, and the other is women. For among men and women, fractions $\omega^e(m), \omega^e(f)$ are college graduates, and the rest are non-college graduates. For each college and non-college graduate, fractions $\omega^m(c), \omega^m(nc)$ are married, and the rest are single. The assortativeness of marriage in educational level is given by $F^m(e_H, e_W)$ (variables so far are invariant over time).

For married, a fraction $\pi^k(1)$ have small kids. At birth, $\mathcal{E} = 1$ (human capital is not depreciated) and $\nu = 0$ (persistent shocks are not realized yet). η (transitory shock) is drawn from the normal distribution.

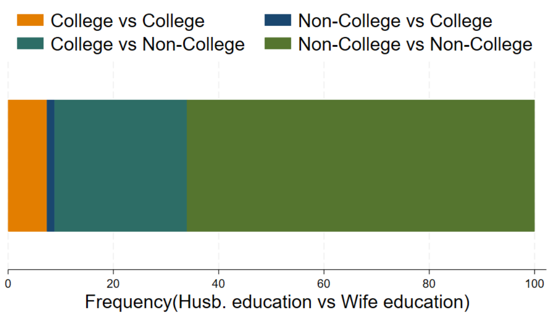
[22] See `https://www.pdrc.keio.ac.jp/en/` for more details or to request the data.



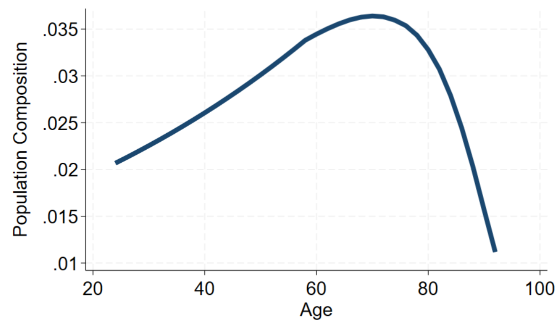
(a) Survival Probability



(b) Childbirth and Child-growth Probability



(c) Assortativeness of Marriage in Education



(d) Population Composition by Age

Figure A2: Other Parameters in Demographics

where Y_{ijc} is the annual earnings of regular male workers aged i in age j and cohort c. $\boldsymbol{D}_{\text{age}}$ and $\boldsymbol{D}_{\text{cohort}}$ are the vectors of age and cohort dummies, respectively, and $\boldsymbol{X}_{ijc}$ is the vector of other covariates containing the difference and the square of the difference of the age between the husband and wife. Using the estimated coefficients, I calculate the wage profile for each age normalizing to the cohort of 1964. For σ_η and σ_ε, size of transitory and permanent productivity shocks, I use the estimates from Lise et al. (2014), which use JPSC and estimate the income process. For σ_Y, the CES parameter of the production function, I use the estimate from Abbott et al. (2019). They estimate the elasticity of substitution between male and female labor in the US. Although the estimate is for the US, since there are no estimates or suitable datasets for Japan, I use it for this study following Kitao and Nakakuni (2023).

The hours worked by a full-time worker $\bar{h}$ is obtained from JPSC. I define hours worked as the sum of working, studying, and commuting hours. I took the average for male regular workers and obtained the fraction of it against the total hours of the week (including weekends). The thresholds of working hours for social securities, 20 hours and 30 hours a week[23] are also obtained from JPSC. I calculate average commuting hours for female non-regular workers and add it to the 20 and 30 hours a week to obtain their thresholds.

C.3 Government

The nominal interest rate r is set to 0.0096, average over 1988-2021, based on BoJ's data.

[23] See section D for more details.

C.4 SMM Moments

The data moments for the SMM estimation are obtained from JPSC. For gender wage gaps within college and non-college graduates, I calculate wages by dividing the annual income before tax by the average hours worked. Using the wage profile, I calculate the average wage by age weighted by working hours controlling for the cohort effects. Then, I calculate the gender gap by

$$\log\left(\bar{w}_j^{\text{male}}\right) - \log\left(\bar{w}_j^{\text{female}}\right),$$

for each age and education group. For the labor force participation rate by age, I calculate the fraction of single and married female workers who answered that they are working. For the consumption ratio between spouses by age, I calculate

$$\log(c_H + 1) - \log(c_W + 1),$$

for couples, and take the average by age. The model moments are calculated by simulating the model and computing the same statistics as the data moments, and the parameters are chosen to minimize the squared errors between the model and data moments normalized by the standard deviation of the data moments.

D Tax System in Japan

This appendix summarizes the tax system in Japan since 1988, focusing on the income tax and local income tax. I assume that the incomes are only wage incomes, and the units of the amounts are all in JPY in thousands.

D.1 National Income Tax

The source of the information in this section is the Ministry of Finance Statistics Monthly.

Basic Deduction All workers in Japan can deduct a certain amount from their annual income before calculating the income tax. The amount of the basic deduction is summarized in Table A1.

Table A1: Basic Deduction (thousand JPY)

Income	1988	1989-1994	1995-2019	2020-
All	330	350	380	—
$\leq 24,000$	—	—	—	480
$\leq 24,500$	—	—	—	320
$\leq 25,000$	—	—	—	160
$> 25,000$	—	—	—	0

Wage Income Deduction The amount of the wage income deduction depends on the annual wage income I as summarized in Table A2.

Table A2: Wage Income Deduction

Income	1988	1989-1994	1995-2012	2013-2015
$\leq 1,425$	570			
$\leq 1,625$		650	650	650
$\leq 1,650$	$40\% \times I$	$40\% \times I$		
$\leq 1,800$			$40\% \times I$	$40\% \times I$
$\leq 3,300$	$165 + 30\% \times I$	$165 + 30\% \times I$		
$\leq 3,600$			$180 + 30\% \times I$	$180 + 30\% \times I$
$\leq 6,000$	$495 + 20\% \times I$	$495 + 20\% \times I$		
$\leq 6,600$			$540 + 20\% \times I$	$540 + 20\% \times I$
$\leq 8,500$				
$\leq 10,000$	$1,095 + 10\% \times I$	$1,095 + 10\% \times I$	$1,200 + 10\% \times I$	$1,200 + 10\% \times I$
$\leq 12,000$				
$\leq 15,000$				$1,700 + 5\% \times I$
Max	$1,595 + 5\% \times I$	$1,595 + 5\% \times I$	$1,700 + 5\% \times I$	$2,450$
Income	2016	2017-2019	2020-	
$\leq 1,425$				
$\leq 1,625$	650	650	550	
$\leq 1,650$				
$\leq 1,800$	$40\% \times I$	$40\% \times I$	$40\% \times I - 100$	
$\leq 3,300$				
$\leq 3,600$	$180 + 30\% \times I$	$180 + 30\% \times I$	$80 + 30\% \times I$	
$\leq 6,000$				
$\leq 6,600$	$540 + 20\% \times I$	$540 + 20\% \times I$	$440 + 20\% \times I$	
$\leq 8,500$			$1,100 + 10\% \times I$	
$\leq 10,000$	$1,200 + 10\% \times I$	$1,200 + 10\% \times I$		
$\leq 12,000$	$1,700 + 5\% \times I$			
$\leq 15,000$				
Max	$2,300$	$2,200$	$1,950$	

Spousal Deduction and Spousal Special Deduction Rigorously, the spousal deduction and spousal special deduction are legally different and introduced in different years. However, since both already existed in 1998 and discussing them separately is economically meaningless, I focus only on their sum. Let I_{prm} be the income of the primary earner of a couple and I_{sec} be the spouse's income. Then, Table A3, Table A4, Table A5, Table A6, and Table A7 show the amount of the sum of the spousal deduction and spousal special deduction for each year.

National Income Tax Rate The national income tax rate is progressive with respect to the income after the deductions (basic deduction and spousal deduction in this paper) and is summarized in Table A8. The percentages represent the marginal tax rate for each income bracket.

Table A3: Sum of Spousal Deduction and Spousal Special Deduction (1988)

I_{sec}	$I_{\text{prm}} \leq 10,105$	$I_{\text{prm}} > 10,105$
≤ 570	495	330
≤ 900	$495 - 0.5 \times (I_{\text{sec}} - 570)$	330
$\leq 1,230$	$330 - (I_{\text{sec}} - 900)$	0
$> 1,230$	0	0

Table A4: Sum of Spousal Deduction and Spousal Special Deduction (1989-1994)

I_{sec}	$I_{\text{prm}} \leq 12,205$	$I_{\text{prm}} > 12,205$
< 700	700	350
< 750	650	350
< 800	600	350
< 850	550	350
< 900	500	350
< 950	450	350
$< 1,000$	400	350
$< 1,050$	350	0
$< 1,100$	300	0
$< 1,150$	250	0
$< 1,200$	200	0
$< 1,250$	150	0
$< 1,300$	100	0
$< 1,350$	50	0
$\geq 1,350$	0	0

Table A5: Sum of Spousal Deduction and Spousal Special Deduction (1995-2003)

I_{sec}	$I_{\text{prm}} \leq 12,315$	$I_{\text{prm}} > 12,315$
< 700	760	380
< 750	710	380
< 800	660	380
< 850	610	380
< 900	560	380
< 950	510	380
$< 1,000$	460	380
$< 1,030$	410	380
$< 1,050$	380	0
$< 1,100$	360	0
$< 1,150$	310	0
$< 1,200$	260	0
$< 1,250$	210	0
$< 1,300$	160	0
$< 1,350$	110	0
$< 1,400$	60	0
$< 1,410$	30	0
$\geq 1,410$	0	0

Table A6: Sum of Spousal Deduction and Spousal Special Deduction (2004-2017)

I_{sec}	$I_{\text{prm}} \leq 12,315$	$I_{\text{prm}} > 12,315$
$< 1,030$	380	380
$< 1,050$	380	0
$< 1,100$	360	0
$< 1,150$	310	0
$< 1,200$	260	0
$< 1,250$	210	0
$< 1,300$	160	0
$< 1,350$	110	0
$< 1,400$	60	0
$< 1,410$	30	0
$\geq 1,410$	0	0

Table A7: Sum of Spousal Deduction and Spousal Special Deduction (2018-)

I_{sec}	$I_{\text{prm}} \leq 11,333$	$I_{\text{prm}} \leq 11,888$	$I_{\text{prm}} \leq 12,315$	$I_{\text{prm}} > 12,315$
$\leq 1,500$	380	260	130	0
$\leq 1,550$	360	240	120	0
$\leq 1,600$	310	210	110	0
$\leq 1,666$	260	180	90	0
$\leq 1,750$	210	140	70	0
$\leq 1,828$	160	110	60	0
$\leq 1,900$	110	80	40	0
$\leq 1,971$	60	40	20	0
$\leq 2,014$	30	20	10	0
$> 2,014$	0	0	0	0

Table A8: National Income Tax Rate

Income	1988	1989-1994	1995-1998	1999-2006	2007-2014	2015-
$\leq 1,950$					5%	5%
$\leq 3,000$	10%	10%				
$\leq 3,300$			10%	10%	10%	10%
$\leq 6,000$	20%	20%				
$\leq 6,950$					20%	20%
$\leq 9,000$			20%	20%	23%	23%
$\leq 10,000$	30%	30%				
$\leq 18,000$			30%	30%	33%	33%
$\leq 20,000$	40%	40%				
$\leq 30,000$			40%			
$\leq 40,000$						40%
$\leq 50,000$	50%					
Max	60%	50%	50%	37%	40%	45%

D.2 Local Income Tax

Local income tax is levied similarly to the national income tax, but the rates and deductions are different. Unlike the national income tax, the local income tax is levied on the last year's income. Hence, the year shown in the tables below is the year of the income, not the year when the tax is levied.

Basic Deduction The basic deduction for local income tax is summarized in Table A9.

Table A9: Basic Deduction (Local)

Income	1988	1989	1990-1993	1994-2019	2020-
All	280	300	310	330	—
$\leq 24,000$	—	—	—	—	430
$\leq 24,500$	—	—	—	—	290
$\leq 25,000$	—	—	—	—	150
$> 25,000$	—	—	—	—	0

Wage Income Deduction The wage income deduction for local income tax is the same as the wage income deduction for national income tax.

Spousal Deduction and Spousal Special Deduction The spousal deduction and spousal special deduction for local income tax are summarized in Table A10, Table A11, Table A12, Table A13, Table A14, Table A15, and Table A16.

Table A10: Sum of Spousal Deduction and Spousal Special Deduction (Local, 1988)

I_{sec}	$I_{\text{prm}} \leq 10,105$	$I_{\text{prm}} > 10,105$
≤ 570	420	280
≤ 900	$420 - 14/33 \times (I_{\text{sec}} - 570)$	280
$\leq 1,065$	$140 - 28/33 \times (I_{\text{sec}} - 900)$	0
$> 1,065$	0	0

Local Income Tax Rate The local income tax rate is levied on the income after the deductions (basic deduction, wage income deduction, and spousal deduction in this paper) and is summarized in Table A17. The local income tax rate is progressive before 2007 and proportional since 2007 with respect to the income after the deductions and is summarized in Table A17.

In addition to the income tax, there exists a per capita tax for local tax as in Table A18 The per capita tax is levied only on those with positive income after the deductions.

Table A18: Per Capita Tax

Year	1998-1994	1995-2012	2013-
Amount	3,200	4,000	5,000

Table A11: Sum of Spousal Deduction and Spousal Special Deduction (Local, 1989)

I_{sec}	$I_{\text{prm}} \leq 12,205$	$I_{\text{prm}} > 12,205$
< 708	600	300
< 766	550	300
< 825	500	300
< 883	450	300
< 941	400	300
$< 1,000$	350	300
$< 1,058$	300	0
$< 1,116$	250	0
$< 1,175$	200	0
$< 1,233$	150	0
$< 1,291$	100	0
$< 1,350$	50	0
$\geq 1,350$	0	0

Table A12: Sum of Spousal Deduction and Spousal Special Deduction (Local, 1990-1993)

I_{sec}	$I_{\text{prm}} \leq 12,205$	$I_{\text{prm}} > 12,205$
< 700	620	310
< 750	610	310
< 800	560	310
< 850	510	310
< 900	460	310
< 950	410	310
$< 1,000$	360	310
$< 1,050$	310	0
$< 1,100$	300	0
$< 1,150$	250	0
$< 1,200$	200	0
$< 1,250$	150	0
$< 1,300$	100	0
$< 1,350$	50	0
$\geq 1,350$	0	0

Table A13: Sum of Spousal Deduction and Spousal Special Deduction (Local, 1994)

I_{sec}	$I_{\text{prm}} \leq 12,205$	$I_{\text{prm}} > 12,205$
< 700	660	330
< 750	630	330
< 800	530	330
< 850	480	330
< 900	430	330
< 950	380	330
$< 1,000$	330	330
$< 1,050$	330	0
$< 1,100$	300	0
$< 1,150$	250	0
$< 1,200$	200	0
$< 1,250$	150	0
$< 1,300$	100	0
$< 1,350$	50	0
$\geq 1,350$	0	0

Table A14: Sum of Spousal Deduction and Spousal Special Deduction (Local, 1995-2003)

I_{sec}	$I_{\text{prm}} \leq 12,315$	$I_{\text{prm}} > 12,315$
< 750	660	330
< 800	610	330
< 850	560	330
< 900	510	330
< 950	460	330
$< 1,000$	410	330
$< 1,030$	360	330
$< 1,100$	330	0
$< 1,150$	300	0
$< 1,200$	250	0
$< 1,250$	200	0
$< 1,300$	150	0
$< 1,350$	100	0
$< 1,400$	50	0
$< 1,410$	30	0
$\geq 1,410$	0	0

Table A15: Sum of Spousal Deduction and Spousal Special Deduction (Local, 2004-2017)

I_{sec}	$I_{\text{prm}} \leq 12,315$	$I_{\text{prm}} > 12,315$
$< 1,030$	330	330
$< 1,100$	330	0
$< 1,150$	310	0
$< 1,200$	260	0
$< 1,250$	210	0
$< 1,300$	160	0
$< 1,350$	110	0
$< 1,400$	60	0
$< 1,410$	30	0
$\geq 1,410$	0	0

Table A16: Sum of Spousal Deduction and Spousal Special Deduction (Local, 2018-)

I_{sec}	$I_{\text{prm}} \leq 11,333$	$I_{\text{prm}} \leq 11,888$	$I_{\text{prm}} \leq 12,315$	$I_{\text{prm}} > 12,315$
$\leq 1,550$	330	220	110	0
$\leq 1,600$	310	210	110	0
$\leq 1,666$	260	180	90	0
$\leq 1,750$	210	140	70	0
$\leq 1,828$	160	110	60	0
$\leq 1,900$	110	80	40	0
$\leq 1,971$	60	40	20	0
$\leq 2,014$	30	20	10	0
$> 2,014$	0	0	0	0

Table A17: Local Income Tax Rate

Income	1988-1989	1990-1993	1994-1997	1998-2005	2006-
$\leq 1,200$	5%				
$\leq 1,600$		5%			
$\leq 2,000$			5%	5%	
$\leq 5,000$	10%				
$\leq 5,500$		10%			
$\leq 7,000$			10%	10%	
Max	15%	15%	15%	13%	10%

D.3 Social Security Contributions

There are three categories of insurees in the social security system for pension and health insurance, as discussed in Section 5.

Category II Insurees Category II insurees are employed full-time workers and part-time workers who work for more than around 3/4 of the full-time work (MHLW). They have to pay for the Employees' Pension plan and health insurance provided by the employer, and are eligible for the Employees' Pension, health insurance, and the National Pension. The pension contribution is proportional to the wage income, and the rate is summarized in Table A19. Precisely, the contribution pension contribution is burdened by the employee and the employer equally; hence, the rate in the table is half of the total contribution rate. In addition, there exist upper bounds for the monthly income levied by the pension contribution, which is summarized in Table A20. Thus, the maximal pension contribution is the product of the rate and the upper bound.

Table A19: Employees' Pension Contribution Rate

Year	1988	1989	1990	1991	1992	1993	1994	1995	1996	1997
Rate	6.20%	6.20%	7.15%	7.25%	7.25%	7.25%	8.25%	8.25%	8.68%	8.68%
Year	1998	1999	2000	2001	2002	2003	2004	2005	2006	2007
Rate	8.68%	8.68%	8.68%	8.68%	8.68%	6.79%	6.97%	7.14%	7.32%	7.50%
Year	2008	2009	2010	2011	2012	2013	2014	2015	2016	2017
Rate	7.67%	7.85%	8.03%	8.21%	8.38%	8.56%	8.74%	8.91%	9.09%	9.15%
Year	2018	2019	2020	2021	2022	2023	2024			
Rate	9.15%	9.15%	9.15%	9.15%	9.15%	9.15%	9.15%			

Table A20: Employees' Pension Contribution Upper Bound (JPY in thousand, for monthly income)

Year	1988	1989	1990	1991	1992	1993	1994	1995	1996	1997
Max	470	530	530	530	530	530	590	590	590	590
Year	1998	1999	2000	2001	2002	2003	2004	2005	2006	2007
Max	590	590	620	620	620	620	620	620	620	620
Year	2008	2009	2010	2011	2012	2013	2014	2015	2016	2017
Max	620	620	620	620	620	620	620	620	620	620
Year	2018	2019	2020	2021	2022	2023	2024			
Max	620	620	650	650	650	650	650			

The health insurance contribution, however, varies across employers. Hence, I refer to the contribution rate of the Japan Health Insurance Association (JHIA) in Table A21. JHIA is the largest health insurance association in Japan, and many employers join it to provide health insurance to their employees. Around 30% of the population is insured by JHIA. More precisely, the insurance contribution consists of the health insurance contribution and the long-term care insurance contribution since 2000, and insurees aged more than 40 years old have to pay the long-term care insurance contribution, too. As in the pension contribution, the health insurance contribution is burdened by the employee and the employer equally; hence, the rate in the table is half of the total contribution rate.

Table A21: Health Insurance Contribution Rate

Year	1988	1989	1990	1991	1992	1993	1994	1995	1996	1997
Health	4.15%	4.15%	4.2%	4.2%	4.1%	4.1%	4.1%	4.1%	4.1%	4.25%
Care	–	–	–	–	–	–	–	–	–	–
Year	1998	1999	2000	2001	2002	2003	2004	2005	2006	2007
Health	4.25%	4.25%	4.25%	4.25%	4.25%	4.1%	4.1%	4.1%	4.1%	4.1%
Care	–	–	0.3%	0.6%	0.6%	0.5%	0.6%	0.6%	0.6%	0.6%
Year	2008	2009	2010	2011	2012	2013	2014	2015	2016	2017
Health	4.1%	4.1%	4.67%	4.75%	5.0%	5.0%	5.0%	5.0%	5.0%	5.0%
Care	0.6%	0.6%	0.8%	0.8%	0.8%	0.8%	0.8%	0.8%	0.8%	0.8%
Year	2018	2019	2020	2021	2022	2023	2024			
Health	5.0%	5.0%	5.0%	5.0%	5.0%	5.0%	5.0%			
Care	0.8%	0.8%	0.9%	0.9%	0.8%	0.9%	0.8%			

Category III Insurees Category III insurees are spouses of Category II insurees whose annual income is less than thresholds (JPY 1.3 million in 2024) and who are not insured by the employer. They are exempted from any pension and health insurance contribution but eligible for the National Pension and the National Health Insurance. The thresholds of annual income for the exemption are summarized in Table A22.

Table A22: Threshold for Category III Insurees (JPY in million)

Year	1988	1989-91	1992	1993-
Threshold	1.0	1.1	1.2	1.3

Category I Insurees Category I insurees are those not classified as Category II or III insurees. They are mainly self-employed, unemployed, or employed part-time workers who work for less than around 3/4 of the full-time work. They have to pay for the National Pension and the National Health Insurance and are eligible for the National Pension and the National Health Insurance. Unlike the Employees' pension plan, the National Pension contribution is lump-sum (Table A23). The National Health Insurance contribution is the combination of the lump-sum and proportional contribution. However, the rates and the systems are different across municipalities, and it is difficult to summarize them. Hence, in this paper, following Kitao and Mikoshiba (2022), I use the contribution rate of JHIA as the National Health Insurance contribution rate for Category I insurees, too. This is not completely accurate, but the contribution between JHIA and the National Health Insurance is not so different, and the difference is not so large in the context of this paper.

Table A23: Annual Contributions for National Pension (JPY in thousand)

Year	1988	1989	1990	1991	1992	1993	1994	1995	1996	1997
Amt	92.4	96	100.8	108	116.4	126	133.2	140.4	147.6	153.6
Year	1998	1999	2000	2001	2002	2003	2004	2005	2006	2007
Amt	159.6	159.6	159.6	159.6	159.6	159.6	159.6	162.96	166.32	169.2
Year	2008	2009	2010	2011	2012	2013	2014	2015	2016	2017
Amt	172.92	175.92	181.2	180.24	179.76	180.48	183	187.08	195.12	197.88
Year	2018	2019	2020	2021	2022	2023	2024			
Amt	196.08	196.92	198.48	199.32	199.08	198.24	203.76			

D.4 Social Security Benefits

The amount of the National Pension benefits does not depend on the income level but on the number of years of the contribution. However, since it is legally mandated to pay the National Pension contribution, I assume the amount of the benefits is common across the population and is JPY 816,000 per year using the value in 2024. The amount of the Employees' Pension benefits depends on the income level and evolves over time (see (28)). I set the coefficient $\kappa^P(j)$ in (28) to 7.125×10^{-3} until 2002 and 5.481×10^{-3} since then, following MHLW.

Since those born in 1964 are 60 years old in 2024, I cannot observe the amount of benefits they receive in their retirement age. Hence, I assume the amount of the benefits will not change over time since 2024 although this is not accurate. Also, I ignore the tax levied on the benefits because it is also not accurate and because my focus is on the working-age generations.

D.5 Eligibility Criteria for Employees' Pension Plan

The eligibility criteria for the Employees' Pension Plan differ across the sample period. Throughout the sample period, those who work for more than around 3/4 of the full-time work (about 30 hours per week) are eligible for the Employees' Pension Plan. Since 2016, the eligibility criteria have been relaxed: those who work 20 hours a week and earn more than JPY 88,000 a month and whose company has more than 500 employees. The criteria were relaxed further in 2020, and the number of employees was reduced to 100.